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AND INFORMATION AND COMMUNICATION TECHNOLOGY
(DEPARTMENT OF ELECTRICAL AND ELECTRONICS
TECHNOLOGY)**

**Optimal Sliding Mode Controller for Position Tracking of
Magnetic Levitation System**

MSc Thesis for the Partial Fulfillment of
Master of Science in Electrical Automation and Control Technology Management

By,

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Optimal Sliding Mode Controller for Position Tracking of Magnetic Levitation System

A Thesis submitted to

**TECHNICAL AND VOCATIONAL TRAINING INSTITUTE (TVTI)
FACULTY OF ELECTRICAL AND ELECTRONICS TECHNOLOGY
AND INFORMATION AND COMMUNICATION TECHNOLOGY
(DEPARTMENT OF ELECTRICAL AND ELECTRONICS
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TECHNOLOGY MANAGEMENT**

By,

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Supervisor,

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August 2022
Addis Ababa, Ethiopia

DECLARATION

I hereby declare that the work which is being presented in this thesis entitled “**Optimal Sliding Mode Controller for Position Tracking of Magnetic Levitation System**” is the original work of my own, has not been presented for a master’s thesis in this or other universities and all sources of materials used for this thesis work have been fully acknowledged.

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**TECHNICAL AND VOCATIONAL TRAINING INSTITUTE (TVTI)
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Thesis on

**Optimal Sliding Mode Controller for Position Tracking of Magnetic
Levitation System**

By,
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ABSTRACT

In this report nonlinear adaptive particle swarm optimization-based gain optimization of sliding mode control system presented under matching model uncertainty and random Gaussian external disturbances. First, a third-order dynamic nonlinear model was created, which includes mechanical (ball position and velocity) and electrical (the current) subsystems with uncertainty in the system parameter and loading mass. Secondly, sliding mode control system for position control system under both matched model uncertainty (internal) and external Gaussian disturbances designed for magnetic levitation position tracking system. Then, Optimal gain of sliding mode control system allocated in such way that integral square error fitness function is minimal under both matching model uncertainty and external disturbances using adaptive particle swarm optimization technique. The suggested control method, which is based on the combination of the proposed sliding mode control system and adaptive particle swarm optimization, offers a control performance with a significant improvement in terms of chattering reduction, high precision control accuracy, fast convergence, along with simple design for practical applications. Moreover, robust optimal SMC controller designed for magnetic levitation system under both internal and external disturbances are able reject all disturbances. The magnitude of performance indices is 0.0056, 0.00055, 0.683 and 0.15206 using ISE, ITSE, IAE and ITAE respectively for constant reference positions respectively and similarly, for constant plus sinusoidal reference position 0.0056 using ITSE. SMC parameters with minimum error used for simulation result analysis i.e., 0.00055 ITSE. Finally, the algorithm's performance is demonstrated by model simulation and the proposed control, with simulation results indicating good convergence for given constant and constant plus sinusoidal reference positions.

Key Words: *Sliding Mode Controller, Magnetic Levitations, adaptive particle swarm optimization, model uncertainty, Gaussian disturbances, arctangent function*

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ABBREVIATIONS

IAE	Integrated Absolute Error
ITAE	Integrated Time Weighted Absolute Error
ISE.....	Integrated Squared Error
ITSE	Integrated Time Weighted Squared Error
PID	Proportional-Integral-Derivative
PSO	Particle Swarm Optimization
SMC	Sliding Mode Control
DDMR.....	Differential Drive Mobile Robot
MLS	Magnetic Levitation System
H_{∞}	H-Infinity
GA.....	Genetic Algorithm
FF	Fruit-Fly
ZN	Ziegler Nichols
FL.....	Feedback Linearization
MPC	Model Predictive Control
VSC.....	Variable Structure Control
LQ	Linear-Quadratic
TSM	Terminal Sliding Mode
MIMO	Multiple-Input Multiple-Output
ISMC.....	Integral Sliding Mode Control
HOSMC	High Order Sliding Mode Control
CASF.....	Continuous Approximation of Sign Function

CHAPTER ONE

INTRODUCTIONS

1.1. Background of Study

Maglev systems are useful in a wide range of applications, including high-speed maglev passenger trains, frictionless bearings, levitating wind tunnel models, isolating sensitive machinery from vibrations, levitating molten metal in induction furnaces, guiding rockets, gyroscopes, contactless melting, magnetic bearings, micro robots, vibration isolation systems, wafer distribution systems, and levitating metal slabs during manufacturing [1],[2],[3],[4],[5]. Magnetic levitation using permanent magnets was ruled out in. Because there is no equilibrium between two permanent magnets because the magnetic force between them is inversely proportional to the square of the distance. Based on the source of the levitation forces, the maglev systems can be categorized as either attracting or repulsive systems. Controlling these kinds of systems is made more challenging by the fact that they are typically open-loop unstable and are characterized by highly nonlinear differential equations. To maintain the floater in the same position, more sophisticated control is needed because of the maglev system's instability and sensitivity to exogenous noise and disturbance inputs. Building highly effective feedback controllers to regulate the levitated object's position is therefore crucial [6],[7].

Numerous works for regulating magnetic levitation systems have been described in the literature in recent years. Numerous methods were used to help stabilize the highly nonlinear system and assure robust performance while also stabilizing the maglev system [8],[9],[10]. This thesis presents work on automatic gain optimization of sliding mode position control system of magnetic levitation system with single magnet system based on adaptive particle swarm optimization. The first dynamic nonlinear dynamic model for a single magnet is developed, together with system parameters, loading mass, and a reliable sliding mode control system for the maglev system under both internal and external disturbances. To achieve the best gain for sliding mode control systems based on ISE, ITSE, IAE, and ITAE, adaptive particle swarm optimization is applied.

1.2. Objectives

1.2.1. General Objective

In this thesis report, magnetic levitation position tracking system using adaptive PSO gain optimization of SMC proposed under both model uncertainty and random Gaussian disturbances.

1.2.2. Specific Objective

- ✚ Deriving nonlinear dynamic model for maglev system.
- ✚ Deriving state space representation of maglev including matching model uncertainty.
- ✚ Design automatic gain optimization of SMC control parameter using adaptive particle swarm optimization based on integral square error fitness function.
- ✚ To validate the result by using MATLAB/Simulink software.

1.3. Statement of Problem

To offer reliable control and improve the performance of the Maglev system, certain key efforts relating to various control systems have been emphasized. But there is current problem in position tracking, efficiency, levitation capacity, controllers and optimization of magnetic levitation system under both internal and external disturbance. Due to above problems, it needs robust controller which able to renders the problems addressed above which increase the efficiency, convergence rate, levitation capacity and precision of control system.

The factor that restricts the application of the maglev system are:

- Lack of nonlinear control system, which incorporates the model uncertainty with high precision control system, because majority of control methods Based on the model linearization technique for the constrained working point, for instance, PID control, precise linearization control, and state feedback linearization control, the tracking performance can rapidly decrease with growing departures from the operational working points.
- Limited position control system and only regulation problem presented without considering more complicated reference trajectory such as sinusoidal reference signal i.e., lacks of tracking control problems.
- Approximate dynamic model used in most of literature which is lack accurate representation of maglev system.

- Lack of fast convergence and precision of control system under internal and external disturbances.
- Conventional sliding mode chattering phenomena is main problems specially in case of external disturbances rejection we need to increase gain which results high frequency switching yielding high oscillations
- Manually tuning of parameters of SMC for fast convergence and high precision control system leads to chattering
- In addition, system parameter changes such as variations in suspending mass and resistance and inductance due to magnetic heating process, i.e., system parameters and loading mass model uncertainty, should be taken into account.

1.4. Scope of the thesis

The mechanical (ball position and Velocity) and electrical (current) subsystems that were developed are included in this thesis report as a third-order model dynamic model. The scope of this thesis report is starting from deriving dynamic modeling of maglev system and analysis dynamic of maglev on MATLAB/Simulink and designing for position tracking control using a sliding mode control system and testing algorithm on MATLAB/SIMULINK. The expected outcome of this thesis are controller proposed in this thesis report will be able improve the position control system in all perspectives for dedicated maglev system under both internal and external disturbances with a fast convergence, high accuracy and significant reduction of chattering problems by including the systems parameters changes such as the variation of suspending mass and the changes of resistance and inductance because of electromagnet heating process which is system parameters and loading mass model uncertainty using nonlinear control system with good dynamic model which able represent the physical system.

1.5. Limitation of thesis

One of the key factors preventing the practical realization of this was the absence of suitable materials for real world applications. Particularly, feedback sensors are challenging to obtain from domestic markets since, they are perceived as luxury items, and no one will pay attention to such study fields. In adequate financial support from government for physical implementation are some main limitations.

1.6. Significance of the research

The magnetic levitation (Maglev) system significantly contributes to the industrial application. Because to its decreased maintenance costs, greater power efficiency, and decreased power consumption. Maglev trains, medical equipment, such as magnetically hung prosthetic heart pumps, and power generation (using wind turbines, for example) are typical uses. Furthermore, this work will lay the foundation where future researchers and students gain experience and motivation, and the university will start building its name as one of the newly established technological research institutes.

1.7. Methodology

To meet the objectives of the thesis, the following methods will be applied. Basically, it consists of literature review, dynamic modelling of maglev system, adaptive PSO base optimal sliding mode controller design, documentation and finally, presentation.

Literature review: in this part, the review of the magnetic levitation control approach will be discussed with respect this work.

Dynamic modelling of DDMR: under this section, dynamic modelling of maglev system derived using including model uncertainty.

Controller design: after dynamic model of magnetic levitation system has been obtained the next part will be SMC controller design for ball position control system.

Simulation and control algorithm testing on MATLAB/SIMULINK: in this step, simulation result of controller test against both disturbances. If the controller design doesn't meet the expected output, controller parameters changed appropriately in order to meet objectives addressed. Here, adaptive particle swarm optimization optimal SMC designed using different performance evaluation techniques such as ISE, IAE, ITSE and ITAE.

Documentation and presentation: finally, research report will have concluded with results and summarized and final documentation will be presented according schedules.

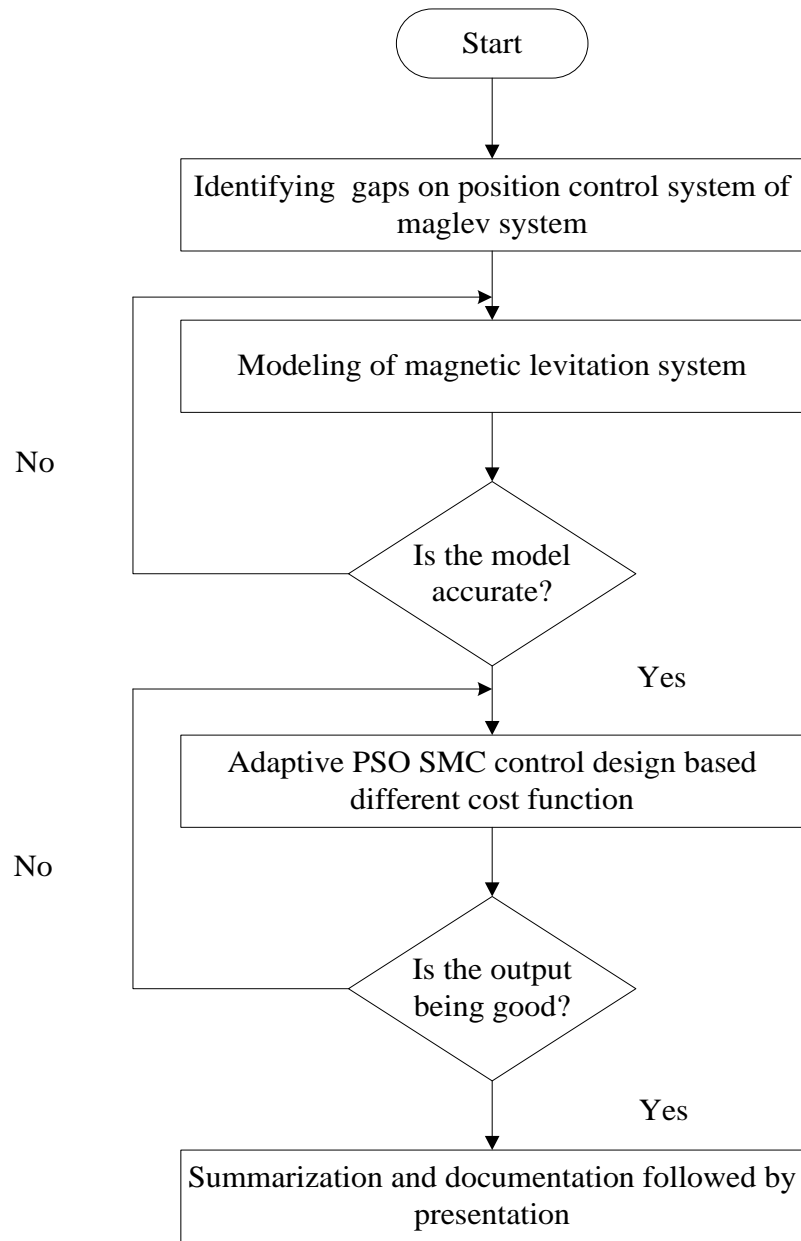


Figure 1. 1 System flowchart

1.8. Outline of the thesis

The rest of the thesis is organized as follows.

In chapter two, maglev system related work, theory with respect SMC, PSO. In chapter three, mathematical model of magnetic levitation is presented with model uncertainty after that, controller is designed for system under chapter four. Depending on the designed controller simulation result and analysis is conducted in chapter five with simulation conducted on MATLAB/SIMULINK. Finally, conclusions and recommendations are drawn in chapter six.

CHAPTER TWO

LITERATURE REVIEW AND THEORETICAL OVERVIEW

2.1. Magnetic levitation control related works

In this chapter, we talk about reviewing the literatures that different academics have already reported on. This section discusses works that are relevant to magnetic modeling and control systems in relation to this thesis report. It ends with a discussion of the improvements to these works. According to [11], state variable feedback control and neural PID controllers are used to design the controllers for the maglev system. Adaptive neural PID magnetic levitation control system is built such that PID control parameters are allocated using back propagation system and contrasted to state feedback control system. Dynamic model of derive and linearized at some operational point. As was said above for position control, the linear model of magnetic levitation uses PID control with a neural system, and the challenge is the regulatory system.

A linear model predictive control technique for a magnetic levitation system is presented in the study suggested [12], (Maglev). The suggested controller is compared to a one-degree freedom PID controller ball position control for a maglev system based on a linear prediction model that was created by linearizing the plant around a known operating point. PID and linear model predictive controllers were tested for controller performance, and a linear model was employed to construct the controller. A new, reliable control of magnetic levitation systems (MLSs) is presented in [13], in this study. The non-singular fast terminal sliding mode is the foundation for the control suggestion. As a result, the suggested system achieves excellent tracking precision, robustness against ambiguous dynamics and outside disturbances, and quick convergence in finite time.

The primary contributions of this study include stability, quick convergence in finite time, and strong tracking performance for the maglev system's approximation dynamic model.

This work's primary flaw is the use of approximations in the controller design, which led to chattering in the controller's output response. A single magnet magnetic levitation technology that is widely used in maglev trains is given [14]. A robust H_∞ control rule based on linear matrix inequality is developed since traditional control laws exhibit poor behavior when coping with unpredictable disturbance signal inputs.

We apply this control law generally to any linear system that receives an unknown disturbance signal. In order to stabilize a magnetic levitation device with a single magnet for disturbance,

rejection, a robust H infinity control technique based on linear matrix inequality is proposed in this study. Poor control performance, including issues with regulation, and less robust.

An adaptive neural terminal sliding mode is used for tracking control of magnetic levitation systems in the presence of dynamical uncertainty and external interruption. The unique fast terminal sliding manifold function is provided together with dynamic coefficients, which enables the system state variables to rapidly converge to the equilibrium point on the manifold function. Additionally, regardless of whether the initial value is close to or far from the sliding manifold, an adaptive, robust reaching control law coupled with a radial basis function neural network compensator drives the system fast approaching the sliding manifold function and reduces chattering of the conventional terminal sliding mode control. The implemented control method offers a control performance with a significant improvement in terms of chattering reduction, high tracking accuracy, fast convergence, along with simple design for practical applications [15]. This is achieved by using a design approach based on the combination of the proposed sliding manifold and the combined control law. Model predictive control of the magnetic levitation system was proposed in [16], the dynamic model of the system was linearized around the operation point, the nonlinear dynamic of the system was derived using the Lagrange formulation, and the ball position control system was used using the model predictive control system. The parameter tuning of the PID controller is carried out by optimizing a suitable performance index based on the ITSE (Integral-time square error) performance criterion using various optimization techniques [17]. This paper proposes a design approach of PID controller based on modern heuristic and intelligent optimization techniques such as GA, PSO, Fruit-fly (FF), and newly introduced Grey Wolf Optimization (GWO). As mentioned above in [17], the dynamic model of the maglev system was linearized and several optimization techniques were used.

PID controller for MLS based on a successful method of Genetic Algorithm (GA) parameter tweaking [18]. For MLS, a PID controller was created. Controller parameters were modified using GA to produce the best performance indices. Results of an analysis of the suggested controller's performance are shown, along with a comparison to Ziegler Nichols, a common PID tuning approach (ZN). PID and metaheuristic optimization were used to construct the linear controller in [17],[18] for the system. The creation and application of a durable, optimal integral sliding mode controller with output feedback for magnetic levitation system position control [19]. In this paper, a fractional order sliding mode controller (FOSMC) for magnetic levitation system is proposed.

Design and analysis of the proposed controller is done based on the linearized mathematical model of Maglev. The model used in this paper takes into account the electrical and mechanical parameters of the system. The FOSMC is designed to achieve stability and robustness towards the matched model perturbations. In this scheme, the fractional order switching surface and order of fractional derivative are obtained using Particle Swarm Optimization (PSO). The fractional order SMC is compared with integer order SMC in terms of settling time and control input chattering.

Table 2. 1 Literature gaps

Ref	Authors	Year	Focus	Gap
1	M.Santhiya and S.Kishore	2020	Adaptive neuro PID position control system	Regulation problem only and used linear model for controller design and low efficiency in terms of convergence and precision
2	Lakshmi Dutta and Dushyant Kumar Das	2020	Linear model predictive control and one degree freedom PID	Linear model for controller design
3	Ngoc Hoai An Nguyen and Anh Tuan Vo*	2020	Terminal sliding mode position controller	Approximate model and chattering problems
4	Haoyue Song, Weiyang Lin*, Maoqiang Zhou, Gang Liu, Huihui Pan, Mingsi Tong	2019	Robust H infinity control of maglev system	Poor efficiency
5	thanh nguyen truong , anh tuan vo , and hee-jun kang	2020	An adaptive neural terminal sliding mode	Approximate dynamic model of maglev system

6	Lafta E. Jumaa Alkurawy, and Khalid G. Mohammed2	2020	Model predictive control	Linear model and regulation ball position problem
7	Soham Dey, Jayati Dey and Subrata Banerjee	2020	Meta heuristic PID optimization for maglev system.	Linear controller
8	Ishtiaq Ahmad , Mohsin Shahzad and Peter Palensky	2014	PID tuning using genetic algorithm	Linear controller
9	Jesvin Jose and Mija S.J.	2020	feedback based robust optimal integral sliding mode controller.	Linear controller and low efficiency
10	Jesvin Jose and Mija S.J.	2020	Particle swarm optimization of FOSMC for magnetic levitation system	a linearized model of the Maglev is considered. FOSMC controller is designed for the Maglev system using the linear model. Inertial weight of PSO is not adaptive and robustness with respect to external disturbance is not indicated. Chattering is not completely reduced.

2.2. Sliding Mode in Control Design

Variable Structure Control (VSC) is a type of control whose main objective is to move system states closer to a sliding surface. One of the most efficient methods for resolving control issues, sliding mode techniques are gaining popularity. Early in the 1950s in the Soviet Union, Emiliano

and a number of significant researchers—including Atkins and Tikiş—developed and elaborated VSC with SMC. Over the past ten years, VSC and SMC have drawn a lot of interest in the field of control research [21].

The most well-known aspect of sliding mode control is that it completely resists parametric uncertainty and outside disturbance[21],[22]. To push a nonlinear plant state trajectory onto a desirable and user-selected surface in the state space known as the slide or switching surface, and to retain it there for all subsequent time, VMC uses a high-speed switching control algorithm. This surface is known as the switching surface because a control path has one gain if the plant state trajectory is "above" the surface and a different gain if the trajectory is "below." The structure of control the system is alters from one to the next through the process, gaining the term VSC. The control is same times known as SMC [23] to stress the importance of the sliding mode. The state trajectory's progression is separated in to two components.

- Reaching phase and
- Sliding phase

2.2.1. Sliding Surface Base Approach

Linear and nonlinear sliding surfaces are the two varieties. The sliding mode differential equation can be defined in a number of ways, some of which are listed below. Think about the situation that follows.

$$x' = A(x) + B(x)u \text{ with sliding surface of } s = \{x | s(x) = 0\} \quad (2.1)$$

Where $A(x), B(x)$ are general nonlinear functions of x and $x \in R^n, u \in R^m$.

Canonic form: If the system model can be translated to controllable canonic form for a linear single-input system is [24]:

$$x'_i = x_{i+1} \quad i = 1, 2, \dots \dots n - 1 \quad (2.2)$$

$$x'_n = -\sum_i^n a_i x_i + b \cdot u \quad (2.3)$$

It is possible to define the sliding surface by:

$$s' = \frac{\partial s}{\partial x} s(x) = \lambda_1 x_1 + \lambda_2 x_2 + \dots \dots + x_n = 0 \quad (2.4)$$

Where $\lambda_i = \text{const}$, $i = 1, 2, \dots, n - 1$. The intended properties of the sliding mode, or the properties of the closed-loop system following the reaching phase, are defined by the coefficients in the switch function (2.3).

If the system (2.1) is linear and described [25].

$$x' = A(x) + B(x)u \quad (2.5)$$

Consider the case where Q is a non-singular Transformation.

$$BQ = \begin{pmatrix} 0 \\ B_2 \end{pmatrix} \quad (2.6)$$

Where is B_2 and nonsingular. After then, the system is turned into

$$x'_1 = A_{11}x_1 + A_{12}x_2 \quad (2.7)$$

Where, $x_1 \in R^{n-m}$, $x_2 \in R^m$.

The switching surface can be expressed in the following way: $s(x) = \Delta_1 x_1 + \Delta_2 x_2$. We can assume, without losing generality, that Δ_2 is nonsingular, and we have a sliding mode $\Delta_1 x_1 + \Delta_2 x_2 = 0$, i.e., x_2 is related linearly to x_1 and the system satisfies

$$\begin{cases} x'_1 = A_{11}x_1 + A_{12}x_2 \\ x'_2 = -kx_1 \end{cases} \quad (2.8)$$

Where, $k = \Delta_2^{-1} \Delta_1$. (2.8) represents an $(n-m)$ the order system in x_2 is regarded as the control input to the restricted system and defines the dynamic behavior of the sliding mode by:

$$x'_1 = [A_{11} - A_{12}k]x_1 \quad (2.9)$$

The procedures described above show how the challenging state feedback design challenge of developing an adequate sliding surface can be reduced to a smaller order. Generally speaking, if (A, B) can be controlled, then (A_{11}, A_{12}) can be controlled as well, making it possible to calculate K using common feedback design techniques like either linear quadratic techniques or pole placement.

$[A_{11} - A_{12}k]$ Possesses desirable features after determining K , the desired switching function can be designed as follows:

$$s(x) = \Delta_2 [k, I] \cdot x \quad (2.10)$$

Where Δ_2 can be selected arbitrarily. A simple selection is to let $\Delta_2 = I$.

The linear-quadratic (LQ) method:

For linear time-invariant systems, the optimal sliding mode, or more precisely, the ideal selection of the vector K of (2.9), may be discovered by minimizing a quadratic cost over an infinite time

period [25]. For instance, LQ optimization may be used to find the ideal sliding mode for (2.9) by reducing its size since x_2 is the system's input (2.9).

$$J = \int_{t_s}^{\infty} (x_1^T Q_{11} + x_1 + 2x_1^T Q_{12} x_2 + x_2^T Q_{22} x_2) dt \quad (2.11)$$

We can let, without sacrificing generality $Q_{12} = 0$, and then the optimal control x_2 is obtained by

$$x_2 = x_2 - Q_{22}^{-1} A_{12}^T P x_1 = -k x_1 \quad (2.12)$$

P is a positive definite matrix that is the Riccati equation's solution

$$A_{11}^T P + p A_{11} - p A_{12} Q_{22}^{-1} A_{12}^T P = -Q_{11} \quad (2.13)$$

The switching function (2.10) is then calculated by:

$$s(x) = k x_1 + x_2 = -[Q_{22}^{-1} A_{12}^T P] \cdot x \quad (2.14)$$

Time-varying surface for tracking control: According to the desired control bandwidth for a single input system, the sliding surface can be defined in one way [26],[27].

$$s(x, t) = \left(\lambda + \frac{d}{dt} \right)^{n-1} x = 0 \quad (2.15)$$

The closed-loop bandwidth is specified by the strictly positive constant, and the tracking error is represented by the letter x . We can observe that X 's tracking inaccuracy is the only factor affecting S . For instance, if n is 2,

$$s = x' + \lambda x \quad (2.16)$$

Which is just a weighted combination of the location and velocity errors; and if $n = 3$,

$$s = x'' + 2 \lambda x' + \lambda^2 x \quad (2.17)$$

The scalar s can also be viewed to reflect a real measure of tracking performance.

Method of equivalent control: Recognize that there is an analogous control that $t s'(x) = 0$ is a necessary condition for the state trajectory to remain on the sliding surface $s(x) = 0$ [28]

As a result, the settings $s'(x) = 0$, i.e

$$s' = \frac{\partial s}{\partial x} x' = \frac{\partial s}{\partial x} (A(x) + B(x)u(t)_{eq}) = 0 \quad (2.18)$$

Solving (2.18), for $u(t)$ eq. yields the equivalent control.

$$u(t)_{eq} = -\frac{\partial s}{\partial x} A(x) \left(\frac{\partial s}{\partial x} B(x) \right)^{-1} \quad (2.19)$$

Where, $\frac{\partial s}{\partial x} B(x)$ is nonsingular,

The dynamic of system governed by:

$$x' = \left(I - B(x) \left(\frac{\partial s}{\partial x} B(x) \right)^{-1} \frac{\partial s}{\partial x} \right) A(x) \quad (2.20)$$

Terminal sliding mode control:

The recent hot subject is the quick convergence of the SMC scheme. Selecting a switching manifold with the necessary dynamic for the state variables limited to the manifold is the aim of the sliding mode control design. Error dynamics, on the other hand, cannot converge to zero in a finite amount of time for conventional switching manifolds since they are often linear hyperplanes with asymptotic stability. Faster error convergence can be achieved by adjusting the sliding mode settings, but doing so will increase the control gain, which could lead to severe chattering on the sliding surface and lower system performance. To achieve quick convergence in the sliding phase, the terminal sliding mode (TSM) control was created based on finite-time control theory [29],[30],[31]. When compared to the SMC design mentioned, one of the most important aids of the TSM concept is announcing finite-time convergence to the sliding phase to complete the global finite-time convergence. Furthermore, terminal SMC techniques were used to obtain high precision control.

The following is a simple summary of the underlying principle of TSM control: Consider a second-order nonlinear dynamic system that is uncertain.

$$\begin{aligned} x'_1 &= x_2 \\ x'_2 &= f(x) + g(x) + b(x)u \end{aligned} \quad (2.21)$$

Where, $x = [x_1, x_2]^T$ is the system state vector, $f(x)$ and $b(x) \neq 0$ are a smooth nonlinear function of x and $g(x)$ symbolizes the unpredictability's and disruptions that must be satisfying $\|g(x)\| \leq l_g$. Where, $l_g > 0$ and the scalar control input is u . The following first-order terminal sliding variable describes the standard TSM:

$$s = x_2 + \beta x_1^{q/p} \quad (2.22)$$

Where, $\beta > 0$ is a design constant, and p and q are positive odd integers that satisfy the following condition: $p > q$ The existence of TSM is a sufficient requirement.

$$\frac{1}{2} \frac{d}{dt} s^2 \leq \mu |s| \quad (2.23)$$

Where $\mu > 0$ is constant. For the system in equation (2.20) and (2.21) the following is an example of a common control design.

$$u = -b(x)^{-1}(f(x) + \beta^{q/p} x_1^{p/q-1} x_2 + (l_g + \mu) \text{sign}(s)) \quad (2.24)$$

Terminal Sliding Mode is ensured as a result of this:

It is clear that if $s(x) \neq 0$, the system states will reach the sliding mode $s = 0$ within the finite time t_r , which satisfies

$$t_r \leq \frac{|s(0)|}{\mu} \quad (2.25)$$

The system dynamics are described by the following nonlinear differential equation when the sliding mode $s = 0$ is reached:

$$x_2 + \beta x_1^{p/q} = 0 \quad (2.26)$$

Where $x_1 = 0$ is the terminal attractor of the system (2.26)

the finite time t_s that is taken to travel from $x_1(t_r) \neq 0$ to $x_1(t_r + t_s) = 0$ is given by

$$t_s = \frac{p}{\beta(p-q)} |x_1(t_r)|^{1-\frac{q}{p}} \quad (2.27)$$

This suggests that both the system states x_1 and x_2 in the TSM manifold (2.27) converge to zero in finite time.

Global fast terminal sliding mode control:

Asymptotical convergence of states is overcome by the standard sliding mode.

Compared to traditional sliding mode control, fast terminal sliding mode control has a higher convergent characteristic. Additionally, the chattering phenomenon is avoided since the global rapid terminal sliding mode control lacks a switch function[32].

A new global fast terminal sliding surface is presented based on the linear and terminal sliding surfaces:

$$s = x' + \alpha x + \beta x^{q/p} \quad (2.28)$$

Where, $x \in R$ is stating, $\alpha, \beta > 0$, p and q ($p > q$) are positive odd numbers [32].

The time interval that the initial state $x(0) \neq 0$ attains at $x=0$ is

$$t_s = \frac{p}{\alpha(p-q)} \ln \frac{\alpha x(0)^{(p-q)/p} + \beta}{\beta} \quad (2.29)$$

By designing α, β, p, q . In a finite amount of time, we can bring the system state to a condition of equilibrium t_s .

From equation (2.28) we have:

$$x' = -\alpha x - \beta x^{p/q} \quad (2.30)$$

When the state x is far away from the origin, the fast terminal attractor determines the convergence time $x' = -\beta x^{p/q}$; when the state x approaches the origin $x = 0$, then, the convergent time is decided by the equation $x' = -\alpha x$. x converges to 0 Exponentially. As a result, the terminal attractor is introduced in to Eq. (2.29) of the sliding surface, causing the state to converge to zero in a finite time. Furthermore, the linear sliding surface's speed is assured. As a result, the state can reach.

2.2.2. Control law/Reaching law/

To do this, pick a state feedback control function (u: Rn Rm) that can guide the state trajectory in that direction and keep it there. In other words, the managed system must meet the achieving requirements. In MIMO systems, many switching techniques employ various reaching rules throughout the approach to the sliding mode. The most often applied reaching laws and suggested control strategies are described in [31] and[33].

- The direct switching approaches
- Approach based on the Lyapunov function
- Gao strategy to enacting legislation

The direct switching function approach:

The classic sufficient condition for the appearance of sliding mode is to meet the criteria.

$$s'_i s_i < 0, i = 1, \dots \dots m \quad (2.31)$$

a similar approach or condition is proposed in [24], i.e.

$$\lim_{s_i \rightarrow 0^+} s'_i < 0 \text{ and } \lim_{s_i \rightarrow 0^-} s'_i > 0 \quad (2.32)$$

individual switching surfaces and their intersections are all sliding surfaces as a result of these reaching principles in a VSC. This reaching is global, but there is no assurance that it will take a certain amount of time.

The technique based on the Lyapunov function:

Selecting and developing a stabilizing control law is necessary to maintain the system states on such a surface. The generalized version of the equation below was proposed by Stoline and Li in [34] for selecting the geometry of the sliding surface. Where x , the control variable or state vector, $t(x)$, the tracking error defined as $X_d - X$, λ , a positive constant interpreting the surface dynamics, and f , the relative degree of the sliding mode controller are present. When the state trajectory hits

the sliding surface, this is the attractiveness condition. The equation below illustrates how to construct the Lyapunov candidate function, a positive scalar function. Select the control law that will cause the system state variables' reduction of this function: There is no standard guideline for Lyapunov function; nonetheless, in this situation, examine the positive Lyapunov candidate function for all single dynamics systems.

Choose the $u(t)_c$ for negative definite of $v(x, t)' < 0$ so that the trajectory converges to the surface, typical choice

$$u(t)_c = u(x) \text{sign}(s) \quad (2.33)$$

Where, $(x) \in R^{m \times m}$.

while locations on individual switching surface may or may not belong to the sliding surface, the ultimate sliding mode is only guaranteed at the intersection of all switching surfaces as a result of this reaching law.

GAO's reaching law:

In [35], GAO and Hung proposed a reaching law that uses a differential equation to directly specify the dynamics of the switching surface. Reaching phase and sliding phase are both part of the sliding mode based on the reaching law. The sliding phase drive system ensures slide to equilibrium, while the reaching phase drive system maintains a steady manifold. The switching variable $s(x)$ to reach the switching manifold S at a consistent rate by this law $|s'_i| = -q_i$. The simplicity of this reaching law is its greatest asset. However, if q_i is too small, the reaching time will be excessively long. A huge q_i , on the other hand, will produce severe chattering.

❖ Attaining a constant plus proportionate rate:

$$s' = -Q \text{sign}(s) - P \cdot s \quad (2.34)$$

When s is big, introducing the proportional rate term $-ps$, forces the state to approach the switching manifolds faster.

❖ Power rate reaching:

$$s' = -p_i |s_i|^\alpha \text{sign}(s_i), 0 < \alpha < 1, i = 1, \dots, m \quad (2.35)$$

When the state is far away from the switching manifold, this reaching law raises the rate, but when the state is close to the manifold, it decreases the rate.

2.3. Adaptive Particle Swarm Optimization

PSO is a well-liked optimization technique and a population-based optimization tool that may be used for a variety of applications. The PSO algorithm was inspired by the swarming behavior of a

group of birds, a school of fish, or a swarm of bees [36]. This approach, which combines simplified social and global features, is shown to be very effective in addressing continuous nonlinear problems. Each particle, or another term known as an agent in the development of PSO technique, will cross the XY coordinates within the two-dimensional searching space. The update of the new position of the particles will take into account the previous velocity and position data. [38]. Every iteration will save the best value that each particle has obtained. Which we'll compare afterwards to get our individual optimum values. The global best value can be calculated between the individual best values [37]. The controller of the system will use and implement the last iteration with the best global best value.

The particles will move to their new position during the iterations in accordance with their respective velocity and position equations (v equation in 2.36 and s equation in 2.37, respectively) [38].

It is used to firstly optimize the continuous space. The PSO may be mathematically expressed in continuous space coordinates as follows. Each particle in a group of m particles considers their search to the best historical points and the search of the other particles in the best historical point inside the group when moving in a D-dimensional search space at a fixed speed. Each particle shifts in position as a result.

$$v_i^{k+1} = \theta v_i^k + c_1 * r_1 * (p_{best,i} - s_i^k) + c_2 * r_i * (g_{best,i} - s_i^k) \quad (2.36)$$

$$s_i^{k+1} = s_i^k + v_i^{k+1} \quad (2.37)$$

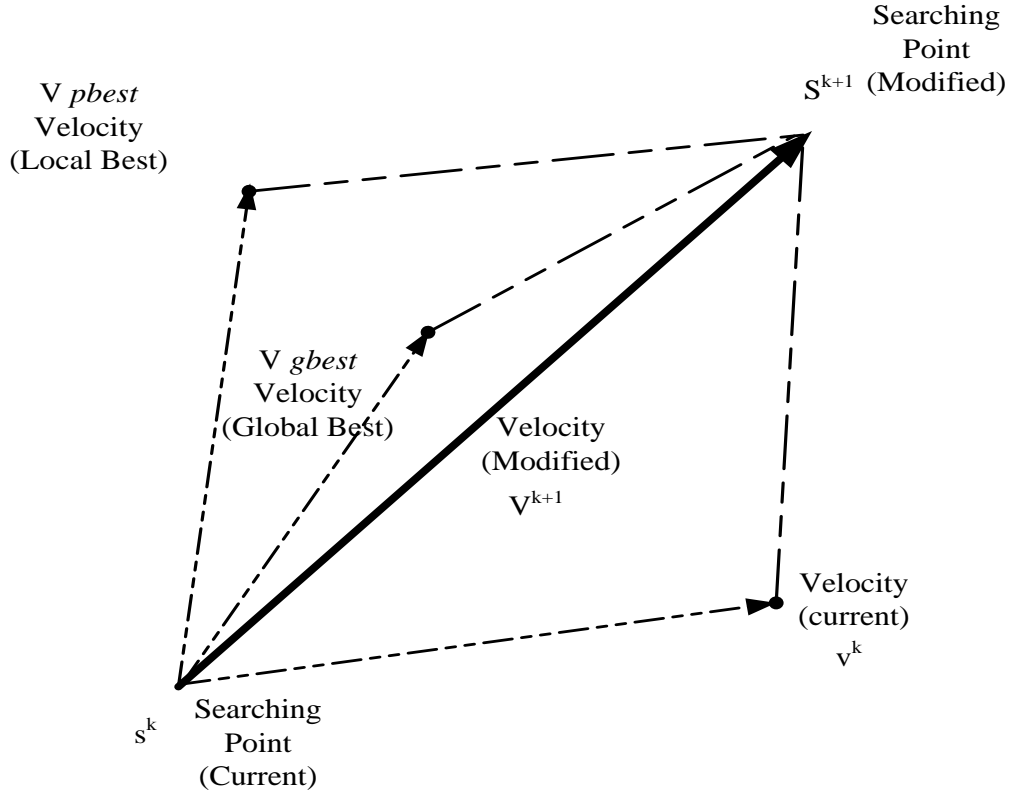


Figure 2. 1 The changes of search point in PSO

Where, s_i^k and s_i^{k+1} are the current and future searching point, v_i^k and v_i^{k+1} denotes the present and future velocity, while the $v_{pbest,i}$ and $v_{gbest,i}$ are the velocity based upon individual best and global best respectively. In order to reaching the new point s_i^{k+1} , present velocity v_i^k , v_i local or individual best velocity $v_{pbest,i}$, and global best velocity $v_{gbest,i}$ will be summarized as in equation (19) to obtain the modified velocity v_i^{k+1} .

The two constants c_1 and c_2 , which are normally equal to one another and are the two non-negative constant accelerations, provide the particles the ability to learn on their own and as a group to perform astonishingly well. The r_1 and r_2 are random numbers between 0 and 1. As a result of checking s_i^{k+1} within the border in the manner described below, we have:

$$\begin{cases} s_i^k > s_{iu}^k, \text{ then } s_i^k = s_{il}^k \\ s_i^k < s_{il}^k, \text{ then } s_i^k = s_{iu}^k \end{cases} \quad (2.42)$$

After evaluation of the above equation (4.17), the objective function is evaluated again, in this thesis, cost functions is given by

$$f_{xi} = \int_0^t e^2(t) dt \quad (2.43)$$

Where, the $e(t)$ is difference between the actual and desired ball position of maglev system.

In order to update the p_{best} and g_{best} , we can use the following criteria such that

$$\begin{cases} f_{xi} \text{ better } f_{p_{best,i}}, \text{ then } s_{best,i}^k = s_i^k \\ f_{p_{best,i}} \text{ better } f_{g_{best}}, \text{ then } g_{best,i}^k = s_{best,i}^k \end{cases} \quad (2.44)$$

The inertia weight is a crucial factor that controls the impact of each particle in the previous velocity. A small inertia weight favors exploitation while the large inertia weight controls the impact of each particle in the previous velocity. The researcher in [39] implement the inertia weight decreased over time using adaptive inertial weight. The purpose in the decline of an inertia weight is to transform the exploratory mode into the exploitative mode that is called adaptive particle swarm optimizations. Here, the inertial weight θ chosed according equation (2.45). The inertia weight function proposed by [39] to control the convergence of the swarm is expressed in equation (2.45).

$$\theta = \theta_{max} - \left(\frac{\theta_{max} - \theta_{min}}{i_{max}} \right) i \quad (2.45)$$

Where the maximum number of iterations is marked by i_{max} , the starting weight coefficient is denoted by θ_{min} , the current iteration is denoted by i and the inertia weighted is denoted by θ .

2.4. Summary of literature review

The papers mentioned above mainly have to do with various models, motion control techniques, and theories of control. Papers addressing position control approaches specifically, whether it can be linear or nonlinear control system and regulation control system or tracking control system depending type of reference used in control system realizations, robustness to internal and external disturbances, convergence rate, modelling of maglev system, control system accuracy, modeling, and chattering problem are therefore presented as in table 2.1 above. An adaptive particle swarm optimization-based gain optimization of sliding mode control system was used in the thesis report to fill in any gaps connected to the aforementioned issues. This system will have robustness to any disturbance namely, external and internal disturbances, high precision, rapid convergence, and a significant reduction in chattering phenomena. Moreover, in this thesis accurate dynamic model derived and nonlinear control system will have perused for both tracking and regulation of ball position using adaptive PSO SMC under both external and internal disturbances. Finally, satisfactory results have been obtained for both constant and constant plus sinusoidal reference positions.

CHAPTER THREE

MATHEMATICAL MODELLING OF MAGLEV SYSTEM

3.1. Dynamics of the Maglev System

The dynamics of a maglev system often unbalanced and the placing of ferromagnetic balls is difficult and requires high accuracy, making a good controller essential [1],[9]. The maglev system maintains a small steel ball in a fixed location at a steady levitation. To sustain the weight of the ball in Fig. 3.1, an electromagnet generates forces. The electromagnetic forces and the electric current that passes through the electromagnetic coil are connected. In most Maglev systems, an electrical loop and an electromechanical loop are present. An analysis of the mechanical and electromagnetic subsystems shapes the maglev system's dynamic attitude.

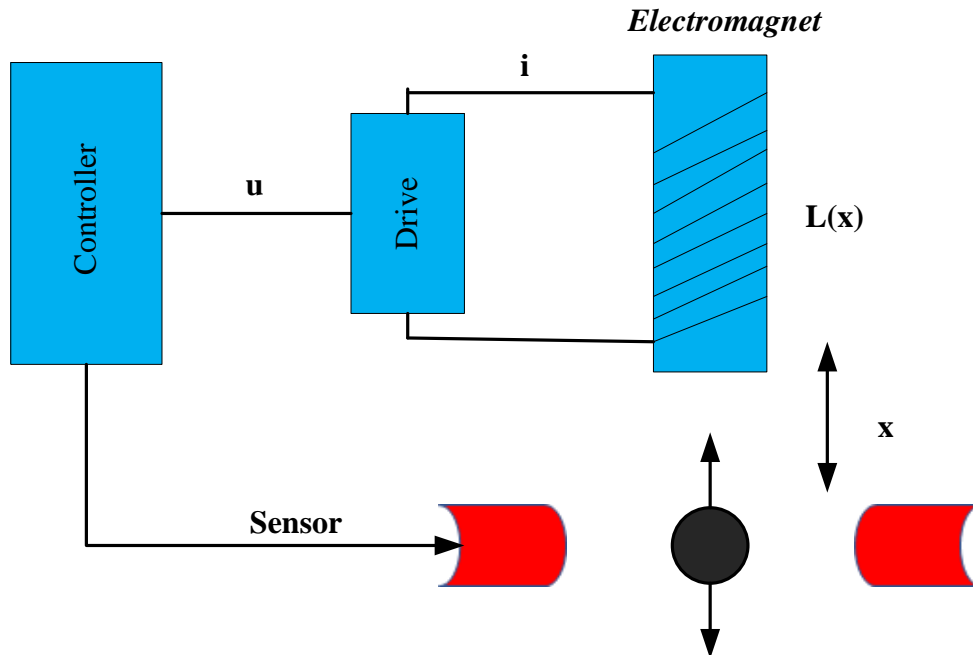


Figure 3. 1 Maglev ball system

Using Kirchoff voltage law, the electromagnetic force is created by the current flowing through the coil:

$$V_{in} = V_R + V_L = iR + \frac{di}{dt}L(x) \quad (3.1)$$

Where V_{in} is the applied voltage, i is the current flowing through the electromagnetic coil, R is the resistance of the coil, and L is the inductance of the coil.

Second, the following model represents a free body notion for a ferromagnetic ball that is perturbed by the electromagnetic force $F_{em}(x, i)$ and the gravitational force F_g . when the ball is perfectly balanced.

$$F_g = F_{em} \quad (3.2)$$

Friction, air force resistance, and other factors are ignored while using Newton's third law of motion.

F_{net} that affects ball is the net force.

$$F_{net} = F_g - F_{em} \quad (3.3)$$

Where g is the gravitational constant, x is the ball's location, m is the mass of the ball, and C is the magnetic force constant.

Finally, for the maglev, the nonlinear model is derived as follows

$$V_{in} = iR + L \frac{di}{dt} \quad (3.4)$$

$$m\ddot{x} = mg - C\left(\frac{i}{x}\right)^2 \quad (3.5)$$

The inductance constraint of the Maglev technique was approximated using a variety of methods.

The inductance L is a nonlinear function of the ball's position x .

$$L(x) = L_c + \frac{lox0}{x} + \dots \quad (3.6)$$

3.2. State Space Representation of Dynamic Model of Maglev

Writing the acquired mathematical model into a state space form simplifies the implementation of control technique [40].

$$x' = f(x, y, u) \quad (3.7)$$

X is State vector; U is control input vector.

$$x = (x, i, v)^T \in R^3 \quad (3.8)$$

Where, the state variable is denoting the ball's position, the ball's velocity, current in the coil of the electromagnet.

State vector can be written as

$$x = (x_1, x_2, x_3)^T \in R^3 \quad (3.9)$$

$$u = (v_{in})^T \in R^1 \quad (3.10)$$

Where, v_{in} are control input voltage of maglev system. The state variable can have assigned to physical variable as follows

$$x = \begin{pmatrix} x_1 = x \\ x_2 = \frac{dx}{dt} \\ x_3 = i \end{pmatrix} \quad (3.11)$$

The consequence of (3.6) substitution with (3.4) is:

$$\begin{aligned} V_{in} &= Ri + \frac{d}{dt} \left(\frac{2c}{x} \right) i + L(x) \frac{di}{dt} \\ &= Ri + L \frac{di}{dt} + 2 \left(\frac{d}{dt} \left(\frac{c}{x} \right) \right) i \\ &= iR + l \frac{di}{dt} - x \left(\frac{i}{x^2} \right) \frac{dx}{dt} \end{aligned} \quad (3.12)$$

Now, let the input control signals and state to be selected as:

$x_1 = x, x_2 = v, x_3 = i, u = v_{in}$, and $x = [x_1 \ x_2 \ x_3]^T$ is the state vector.

Therefore, as state space type, the model of the Maglev system is as follows:

$$\begin{cases} \frac{dx_1}{dt} = x_2 \\ \frac{dx_2}{dt} = g - \frac{C}{m} \left(\frac{x_3}{x_2} \right)^2 \\ \frac{dx_3}{dt} = \frac{-R}{L} x_3 - \frac{2C}{L} \left(\frac{x_2 x_3}{x_1^2} \right) + \frac{1}{L} u \end{cases} \quad (3.13)$$

Allowing

$x^T = [x_1 \ x_2 \ x_3] = [x \ \dot{x} \ i]$, $a_1 = \frac{c}{m}$, $a_2 = \frac{R}{L}$, $a_3 = \frac{2c}{L}$, and $b = \frac{1}{L}$, equations (3.13) can be rewritten as follows:

$$\dot{x} = \begin{bmatrix} x_2 \\ g - a_1 x_3^2 x_1^{-2} \\ -a_2 x_3 + a_3 x_2 x_3 x_1^{-2} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ b \end{bmatrix} u \quad (3.14)$$

Considering the system uncertainties due to deviations of the system parameters and loading mass, equation (3.14) is rewritten in the form of equation (3.15)

$$\dot{x} = \begin{bmatrix} x_2 \\ g - a_1 x_3^2 x_1^{-2} \\ -a_2 x_3 + a_3 x_2 x_3 x_1^{-2} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ b \end{bmatrix} u + \begin{bmatrix} 0 \\ d_1 \\ d_2 \end{bmatrix} \quad (3.15)$$

Where $d_1 = \Delta a_1 x_3^2 x_1^{-2}$ and $d_2 = \Delta a_2 x_3 + \Delta a_3 x_2 x_3 x_1^{-2} + \Delta b u$ in which Δa_1 , Δa_2 , Δa_3 and Δb denote the deviations of a_1 , a_2 , a_3 and b respectively.

CHAPTER FOUR

CONTROLLER DESIGN

4.1. Optimal sliding mode controller for magnetic levitation system

In this thesis, ball position control system using optimal sliding mode control system proposed to keep the ball at some desired position. Position control system designed using third order sliding surface with Gao's reaching techniques and controller parameters of sliding mode controller choose using particle swarm optimization techniques by minimizing different cost function such that error between the actual ball position and desired ball position to be zero. The entire block of diagram optimal sliding mode control system for maglev system is given in the Fig.4.1. below.

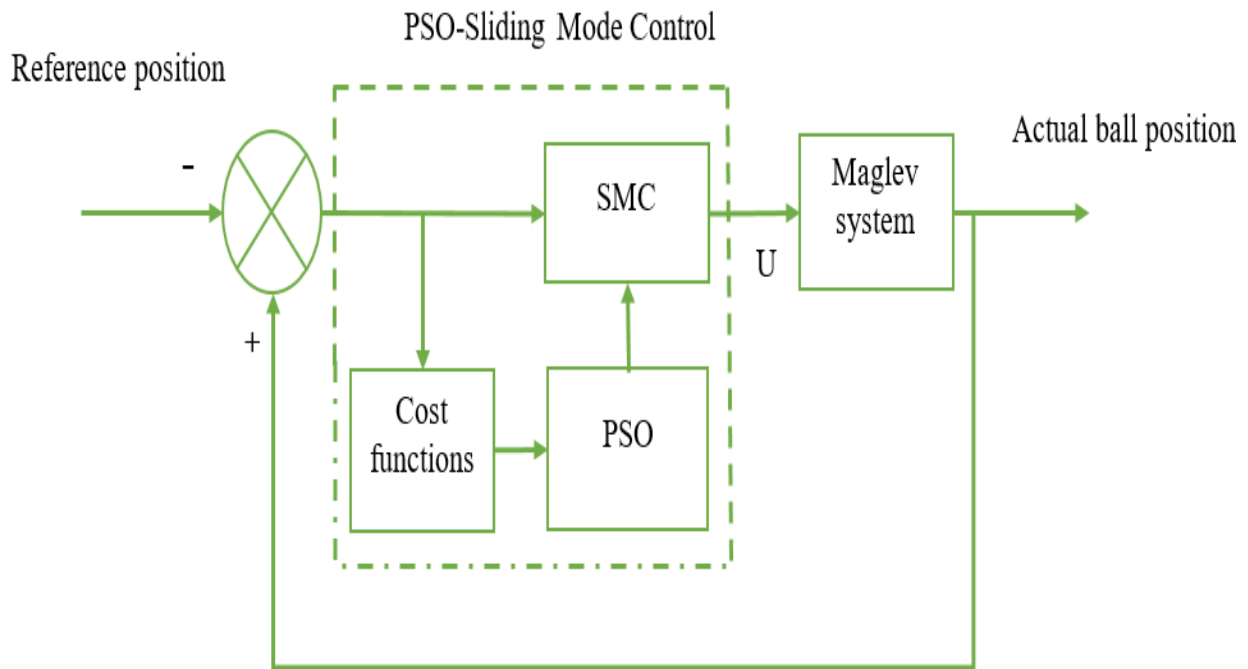


Figure 4. 1 Entire control system for maglev system

Appropriate sliding mode controller parameters are obtained by reducing chattering effects using quasi sliding mode in place of sign function switching function and coefficients of quasi sliding mode choose such way to have less chattering with better performances. The control system performances tested against both matched model uncertainty(internal) and external disturbances. The above block diagram basically is gain particle swarm of optimization of sliding mode control system for maglev systems.

4.2. Sliding Mode Controller Design for ball position tracking

The dynamics of the maglev system can be expressed in new coordinate system as follows

$$\begin{cases} y_1 = x_1 - x_{1d} \\ y_2 = x_2 \\ y_3 = g - \frac{C}{m} \left(\frac{x_3}{x_1} \right)^2 \end{cases} \quad (4.1)$$

The dynamic model of the magnetic levitation system in the new coordinates system can be written as

$$\begin{cases} y_1' = y_2 \\ y_2' = y_3 \\ y_3' = f_1(x) + g_1(x)u \end{cases} \quad (4.2)$$

Where, $f_1(x) = \frac{2C}{m} \left(\left(1 - \frac{2C}{Lx_1} \right) \frac{x_2 x_3^2}{x_1^3} + \frac{R}{L} \left(\frac{x_3}{x_1} \right)^2 \right)$, $g_1(x) = -\frac{2Cx_3}{Lmx_1^2}$

The ball position tracking system output will be expressed as follows

$$w = y_1 = x_1 - x_{1d} \quad (4.3)$$

The third order dynamic model of the system by using state variable can be expressed as form equation above.

$$w''' = f_1(x) + g_1(x)u \quad (4.4)$$

The sliding surface for given system is given by

$$s(x, t) = \left(\lambda + \frac{d}{dt} \right)^{n-1} x = 0 \quad (4.5)$$

The first step in designing an SMC scheme for the system is to design the switching surface. For $n = 3$, weighted sliding surface third order system form the above equation is given by

$$s = x'' + 2\lambda x' + \lambda^2 x = x'' + k_1 x' + k_2 x \quad (4.6)$$

Where, k_1 and k_2 are positives constants.

The sliding mode control system based on gaos reaching law is given by

$$\dot{s} = -\eta \text{sign}(s) \quad (4.7)$$

Where, $\eta \geq |\rho|$, ρ is respresents both internal (model uncertainty)and external disturbances used to dominate the effects both disturbances.

Specifically, sliding surface for maglev system is given by

$$s = w'' + k_1 w' + k_2 w = y_3 + k_1 y_2 + k_2 y_1 \quad (4.8)$$

The above equation (4.8), can be further as

$$s = g - \frac{C}{m} \left(\frac{x_3}{x_1} \right)^2 + k_1 x_2 + k_2 x_1 \quad (4.9)$$

The magnitude control input voltage required to achieve ball position control system is obtained by taking first order derivative above equation and equation with gao's reaching techniques

$$\dot{s} = g - \frac{C}{m} \left(\frac{x_3}{x_1} \right)^2 + k_1 x_2 + k_2 x_1 = -\eta \operatorname{sign}(s) \quad (4.10)$$

The control input extracted form the above equation (4.10), by substituting dynamic of maglev system into above equation (4.10) and (4.9), we get

$$f_1(x) + g_1(x)u + k_1 y_3 + k_2 y_2 = -\eta \operatorname{sign} \left(g - \frac{C}{m} \left(\frac{x_3}{x_1} \right)^2 + k_1 x_2 + k_2 x_1 \right) \quad (4.11)$$

Therefore, from the above equation, the rearranging control input u is given by

$$u = \frac{\left(-\eta \operatorname{sign} \left(g - \frac{C}{m} \left(\frac{x_3}{x_1} \right)^2 + k_1 x_2 + k_2 x_1 \right) - k_1 y_3 - k_2 y_2 - f_1(x) \right)}{g_1(x)} \quad (4.12)$$

Where, $f_1(x) = \frac{2C}{m} \left(\left(1 - \frac{2C}{Lx_1} \right) \frac{x_2 x_3^2}{x_1^3} + \frac{R}{L} \left(\frac{x_3}{x_1} \right)^2 \right)$ and $g_1(x) = -\frac{2Cx_3}{Lmx_1^2}$.

The above equation (4.12) further modified as follows

$$u = \frac{\left(-\eta \operatorname{sign} \left(g - \frac{C}{m} \left(\frac{x_3}{x_1} \right)^2 + k_1 x_2 + k_2 (x_1 - x_{1d}) \right) - k_1 \left(g - \frac{C}{m} \left(\frac{x_3}{x_1} \right)^2 \right) - k_2 x_2 - f_1(x) \right)}{g_1(x)} \quad (4.13)$$

Since, $y_3 = g - \frac{C}{m} \left(\frac{x_3}{x_1} \right)^2$, $y_2 = x_2$ and $y_1 = x_1 - x_{1d}$

4.3. Adaptive Particle Swarm Optimization

In this section specifically, parameters of SMC are optimized by using particle swarm optimization-based for three control parameters namely, k_1, k_2, η in equation (4.13). The objective function values of each particle's placements are evaluated using the PSO technique, which involves inserting a number of fundamental particles into a search space issue. Similar to a cooperative flock of birds looking for food, the particle swarm will eventually concentrate on the goal function that delivers the most rewards. The particle swarm approach is officially expressed as follows in continuous space coordinates: Each particle in the group of m particles in the D -dimensional search space moves at a certain speed while doing the best search feasible. shifting

locations as a result of historical points and the best historical points of the other particles in the group.

The three D-dimensional vectors that make up particle swarm particle i are divided into these three parts:

Current location: $x_i = (x_{i1}, x_{i2}, x_{i3}, \dots, x_{iD})$;

History optimal position: $p_{best,i} = (p_{i1}, p_{i2}, p_{i3}, \dots, p_{iD})$

Velocity: $v_i = (v_{i1}, v_{i2}, v_{i3}, \dots, v_{iD})$;

Here, $i = 1, 2 \dots n$.

The method treats the current location X as a matter of solution for each iteration, viewing it as a collection of coordinates used to characterize the spatial positions. If the present position is superior to the optimal position P , the target position's coordinates are dependent on the current position's existence. Additionally, when the swarm searches for the optimal P , the target location's coordinates are determined depending on the presence of the current position. Additionally, it was noticed that the swarm was now trying to find the ideal place as follows:

$$g_{best,i} = (g_{best,1}, g_{best,2}, g_{best,3}, \dots, g_{best,D}) \quad (4.14)$$

In generally, objective function (cost function), upper (u_b) and lower bound (u_l), number of particles (N), inertial weight (θ), individual and social cognitive (β_1 and β_2) and the number of iterations is the input for particle swarm optimization techniques. Initialization of random position x and velocity within the search space boundary and assign p_{id} and p_{gd} that is individual best and global best based on objective functions.

For each particle, its D-dimensional change for velocity and position according to the following equation:

$$\theta = \theta_{max} - \left(\frac{\theta_{max} - \theta_{min}}{i_{max}} \right) i \quad (4.15)$$

$$v_i(t) = \theta v_i(t-1) + \beta_1 * r_1 * (p_{best,i} - x_i(t-1)) + \beta_2 * r_2 \quad (4.16)$$

$$* (g_{best} - x_i(t-1))$$

$$x_i(t) = v_i(t) + x_i(t-1) \quad (4.17)$$

The two non-negative constant accelerations, β_1 and β_2 , provide particles the potential to summarize themselves and learn on their own while yet contributing to an excellent group. $\beta_1 =$

$\beta_2 = 2$. The r_2 and r_1 are is a random number in the range of 0 to 1. Then, check $x_i(t)$ within boundary using the following method, we have:

$$\begin{cases} x_i(t) > x_u, \text{ then } x_i(t) = x_u \\ x_i(t) < x_l, \text{ then } x_i(t) = x_l \end{cases} \quad (4.18)$$

After evaluation of the above equation (4.20), the objective function is evaluated again, in this thesis, cost functions is given by

$$f_{xi} = \int_0^t e^2(t)dt \quad (4.19)$$

Where, the $e(t) = x_{i1}(t) - x_{id}(t)$ is difference between the actual and desired ball position of maglev system.

In order to update the p_{best} and g_{best} , we can use the following criteria such that

$$\begin{cases} f_{xi} \text{ better } f_{p_{best,i}}, \text{ then } p_{best,i} = x_i \\ f_{p_{best,i}} \text{ better } f_{g_{best}}, \text{ then } g_{best} = p_{best,i} \end{cases} \quad (4.20)$$

Consequently, the following is the particle swarm optimization process:

1. Initialization. the position and speed of particles are created at random in the D-dimensional problem space;
 - objective function, upper (u_b)
 - lower bound (u_l)
 - number of particles (N),
 - inertial weight (θ)
 - individual and social cognitive (β_1 and β_2) and
 - The number of iterations
2. Evaluation of a particle. Examine each particle's optimal update by evaluating the D-dimensional optimization function's performance.
 - If the applicable value is greater than the particle's individual ideal value, the particle is in its present location.
 - The relevant value of the particle is compared to the ideal value over all populations. The particle is where it is now if the current value is better than the best value obtained across all populations. particle updates Modify the particle's location and speed in accordance with formula (4.20).

3. Condition of stopping. Repeat step 2 as necessary to satisfy the termination condition, which is frequently met by the right values and the maximum number of repetitions.

The above procedures are more elaborated using the following flowchart for maglev system ball position control system using particle swarm optimization-based sliding mode control system for three control parameters namely, k_1, k_2, η .

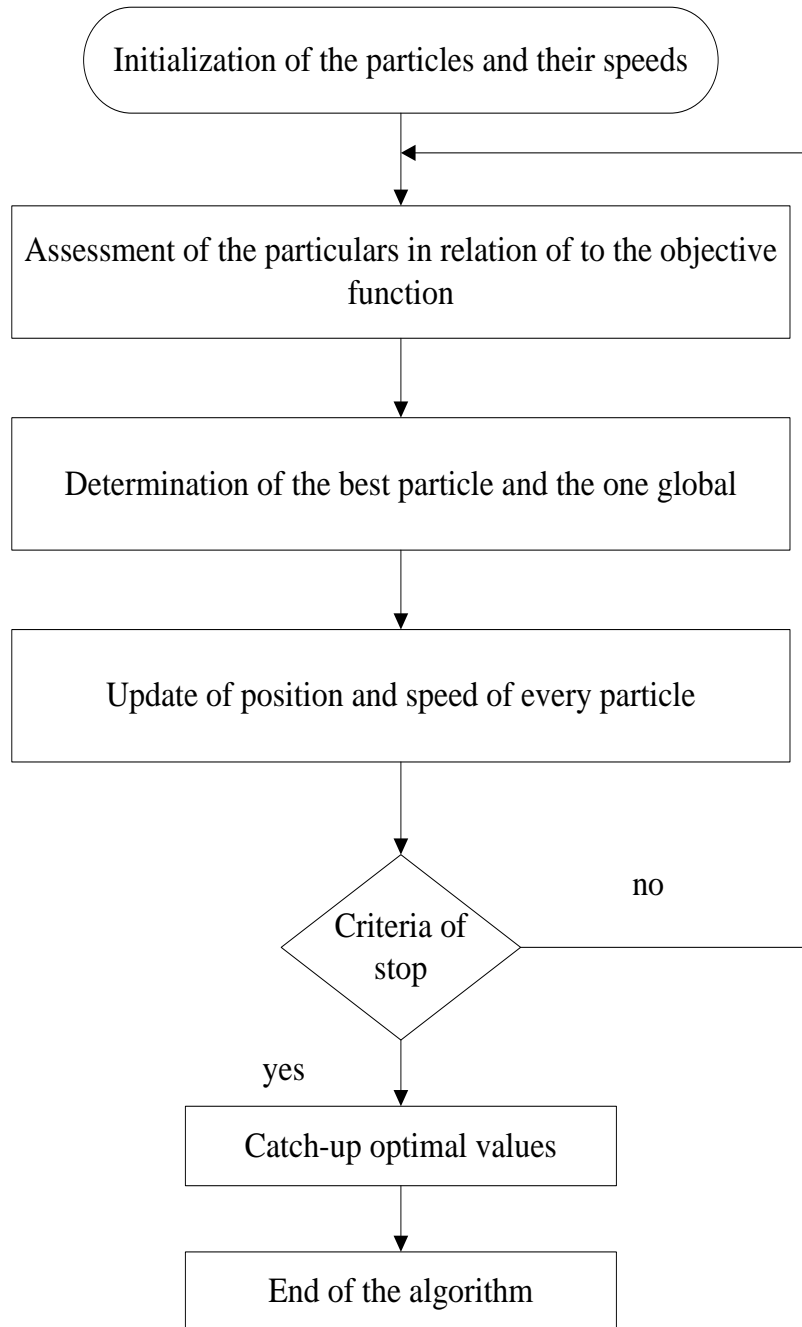


Figure 4. 2 Flowchart of PSO algorithm

4.4. Chattering Reduction

Sliding mode control chattering in real-world engineering systems may harm system elements like actuators. Using the quasi-sliding mode method, which can keep the state in a specific range at a neighborhood, is one option to reduce chattering. The border layer is a term we use frequently. There are common techniques used in continuous systems, which are discussed below [41].

A. Methods for solving the problem of chattering phenomena

- I. Using the tangent hyperbolic function ($\tanh(s/\phi)$) and the deployment of a boundary layer
- II. Using the continuous approximations of the sign function ($\text{sign}(s)$) instead of the discontinuous sign function
- III. Using the saturation function ($\text{sat}(s,\phi)$) instead of the sign function ($\text{sign}(s)$) and deployment of a boundary layer
- IV. Using the reduction of the sign function domain algorithm
- V. Using Integral Sliding Mode Control (ISMC) algorithm
- VI. Using the High-Order Sliding Mode Control (HOSMC) algorithm and etc.

B. Continuous approximation functions of dis-continuous sign function

One suggestion focused on removing undesirable chattering phenomena is to swap out the discontinuous sign function for a Continuous Approximation of Sign Function (CASF). For this substitution, a number of functions have been put forth thus far [21].

1. In order reduce chattering problems relay function (quasi sliding) instead of sgn function

$$\text{sign}(s) = \frac{s}{|s| + \delta} \quad (4.21)$$

Where, $\delta = \varepsilon_0 + \varepsilon_1|x_1 - x_{1d}|$.

2. Saturation function instead of sgn function

$$\text{sat}(s) = \begin{cases} 1, & \text{for } s > \rho \\ \frac{s}{\rho}, & \text{for } |s| \leq \rho \\ -1, & \text{for } s < -\rho \end{cases} \quad (4.22)$$

Where Δ is called “the boundary layer” which is shown in Fig. 4.3. Outside the boundary layer we use switch control and inside the boundary layer we use linear feedback control.

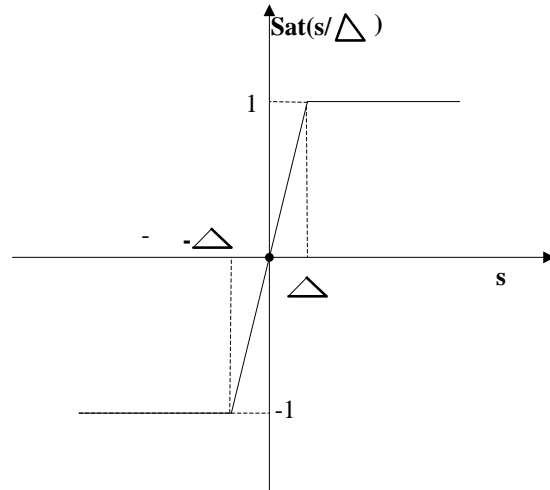


Figure 4. 3 Saturation functions

3 . The system's response will exhibit unwanted chattering due to the discontinuous period. The signum function was replaced with the continuous arctan function to significantly reduce chattering;

$$\text{sign}(s) = \tan^{-1}(s * p) \quad (4.23)$$

Where, $p > 1$.

CHAPTER FIVE

SIMULATION AND RESULT ANALYSIS

5.1. Open Loop Analysis

In this section, open loop dynamic response seen by giving AC voltage source of $v(t) = 25 * \sin(2 * \pi * 50)$ as input voltage in dynamic magnetic levitation system. Specifically, how the position, velocity and current through circuit behave will be analyzed using MATLAB Simulink before applying controller.

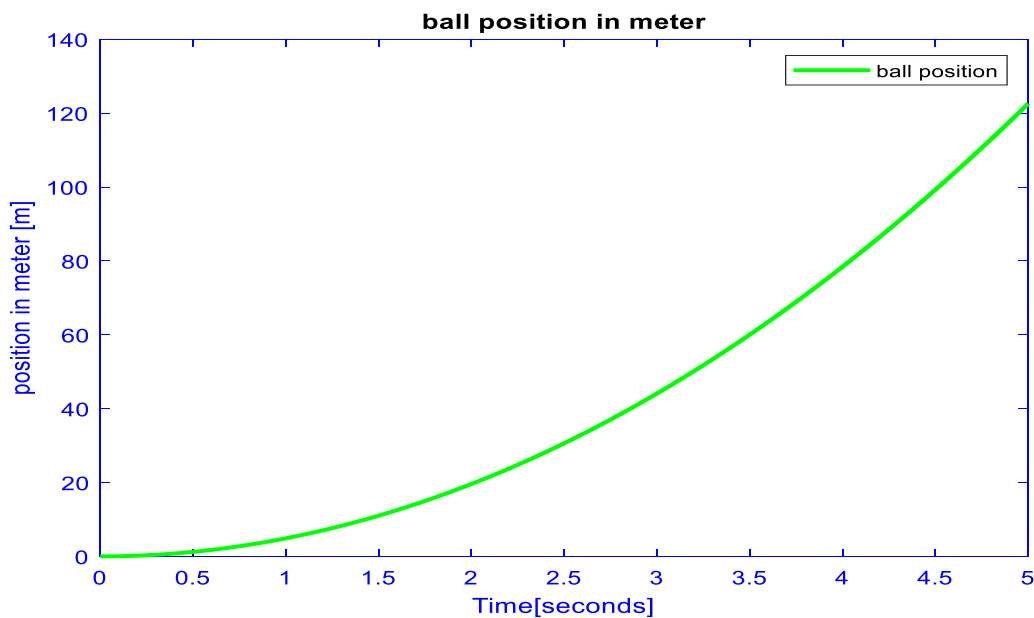


Figure 5. 1 Ball position

As see from Fig 5.1, ball position with respect the applied voltage which increase starting from zero. Moreover, in order maintain ball position at specified reference we need to control vary the supply voltage so that ball position will be at that point. Similarly, Fig 5.2 implies how the velocity of ball look like with respect excitation of input voltage which is increase linearly starting zero in another word its accelerating in forward direction. However, current through maglev system illustrated in Fig 5.3.

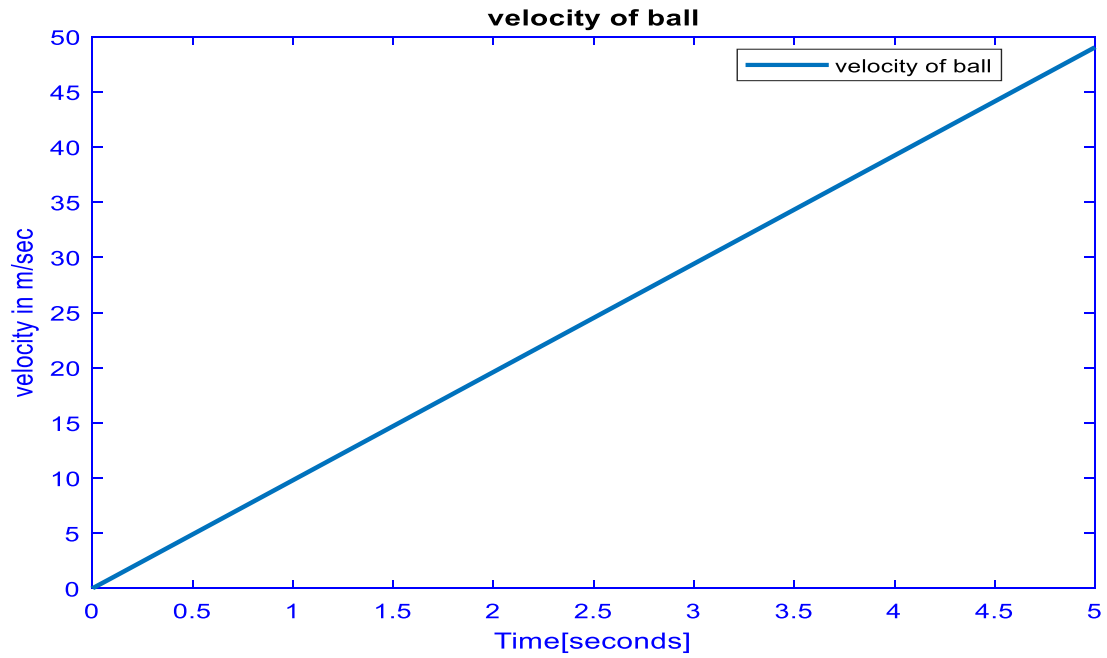


Figure 5. 2 Velocity of ball

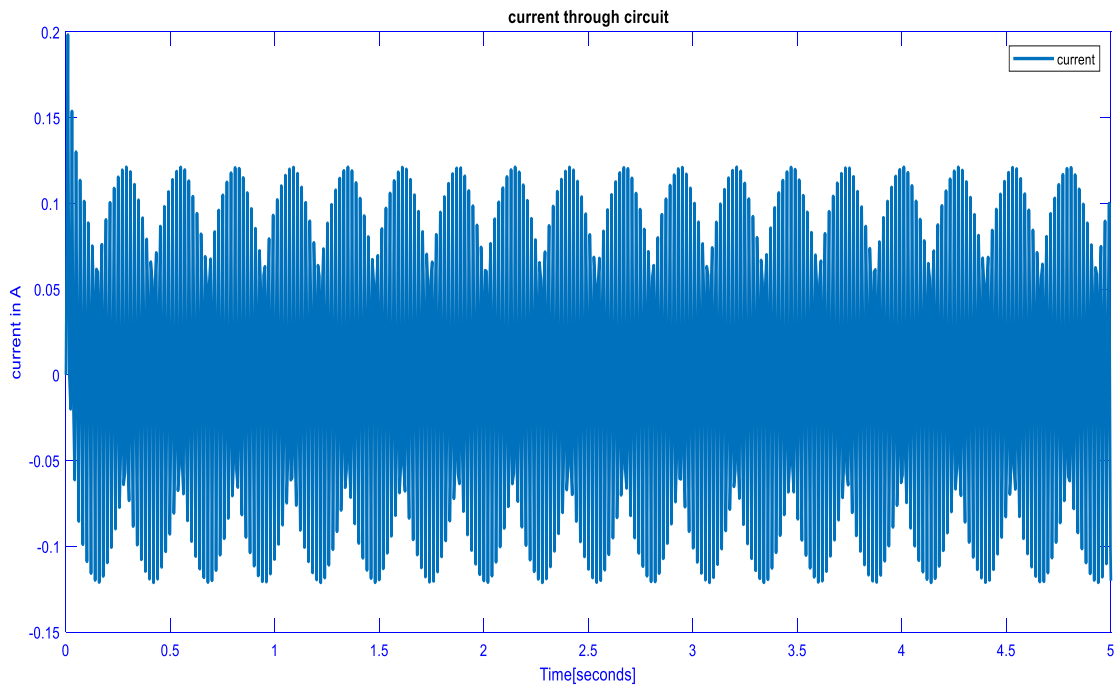


Figure 5. 3 Current through maglev system

5.1.1. Simulation Result of the ball position control system

In this section, the suggested controller approach for the maglev is put into practice in order to simulate the system's stability and validity. The application MATLAB was chosen to simulate the suggested control and delineate the motion of the maglev system. In this section, using computer simulation in MATLAB/SIMULINK, we assess the proposed controller's ability to stabilize the maglev by taking into account various time parameterized reference signals for position control system. First, the effectiveness of the controller is tested in the absence of both model uncertainty and external disturbance, and then the effectiveness of automated gain setting is assessed in the presence of both disturbances. Figure 5.4 depicts the complete Simulink diagram for the magnetic levitation system's sliding mode control system. Sliding mode control systems are typically created for magnetic levitation, therefore then particle swarm optimization applied to automatically select parameters of sliding mode controller.

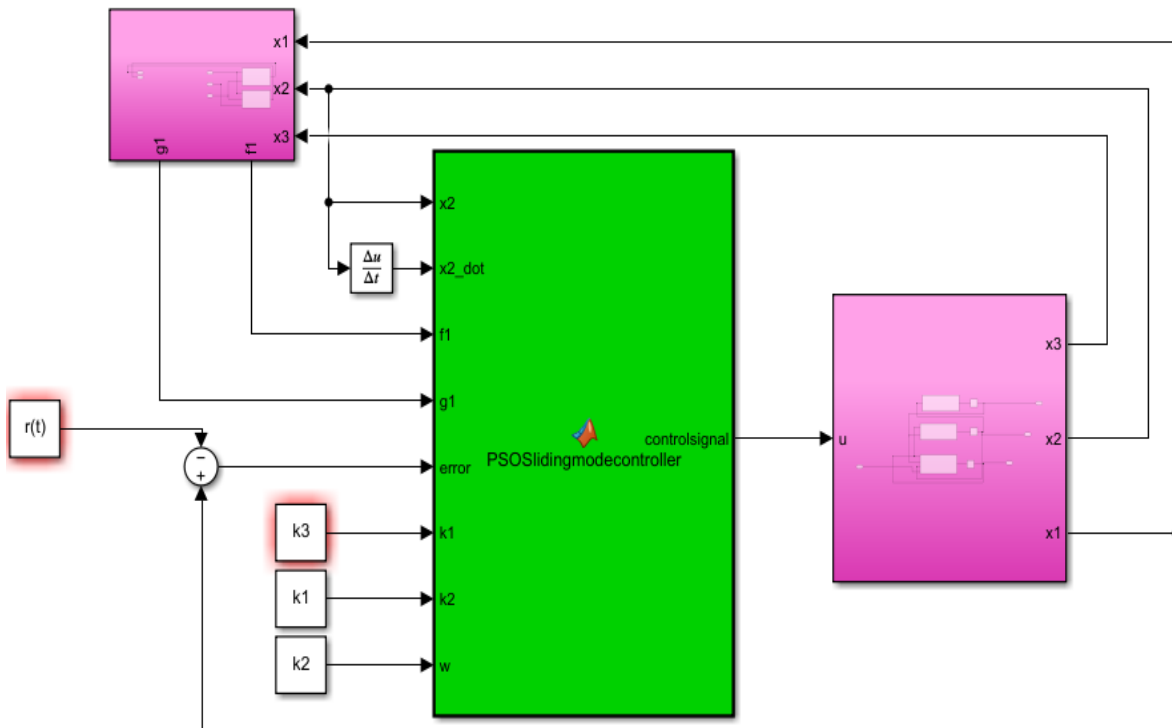


Figure 5. 4 Simulink block diagram for overall maglev system ball position control system

Table 5. 1 Numerical Parameter value of maglev system used for simulation [6]

Parameter	Value and unit
Mass of ball	11.87g

Coils resistance	28.7Ω
Coil inductance	0.65H
Gravitational constant	9.81m/sec ²
Magnetic force constant	0.000141

In this section, we have seen in detail that the proposed control system for tracking performance by giving desired trajectories to states. The following figures shows the performance of the optimal sliding mode control using a particle swarm optimization techniques controller performance for each of all states independently at the same time. In this thesis, the reference ball position selected for simulation purpose are,

$$x_1(t) = 15mm \text{ and}$$

$$x_2(t) = (15 + 15 \sin(0.2 * t))mm$$

The parameters of controller for design of sliding mode control for maglev system is selected using particle swarm optimization techniques such that integral square error is minimized using PSO as follows.

Table 5. 2 Specification of Parameters PSO

Particle swarm particle parameters
$\theta_{max} = 0.9, \theta_{min} = 0.1, \beta_1 = \beta_2 = 2$
Number of particles =20
Number of iterations=200
$x_{u1} = 30000, x_{l1} = 0$
$x_{u2} = 30000, x_{l2} = 0$
$x_{u3} = 500, x_{l3} = 475$
$v_0 = 0$
Initial position selected random between upper and lower bounds

5.1.2. Ball position tracking under no matching model uncertainty and external disturbances

In this section, first of optimal gain of sliding mode control system is obtained using particle swarm optimization mechanism so that error between the actual and reference ball position is minimal using integral square error fitness function under no both external and internal disturbances. After algorithm is executed on MATLAB, the SMC control parameters is obtained. The SMC control parameters for $x_1(t) = 15\text{mm}$ will be given in table 5.3, below as per specification given above.

Table 5. 3 Optimization output using adaptive PSO

Parameter	ISE based PSO	ITSE based PSO	IAE based PSO	ITAE based PSO	Manual tuning
k_1	475	475	475	475	61
k_2	30000	30000	3000	30000	930
η	1922.2	2488.3	1140.1	798.9	350
Error	0.0056	0.00055	0.683	0.15206	0.01

The performances index is given by, $ISE = \sum_1^n e[n]^2$, $ITSE = \sum_1^n ne[n]^2$, $IAE = \sum_1^n |e[n]|$ and $ITAE = \sum_1^n |n * e[n]|$. Where, $e(t) = x_1 - x_{1d}$ is difference between actual and desired ball positions. As seen from Table 5.3 above ball position error is minimum when cost function used for particle swarm is integral time square error with magnitude of 0.00055 and error was maximum under integral absolute error with magnitude of 0.683. In order to reduce chattering phenomena, arc tangential continuous function used which given by $sign(s) = \tan^{-1}(s * p)$. Where, $p > 0$, In this project, the magnitude $p = 10$ used to reduce chattering problems.

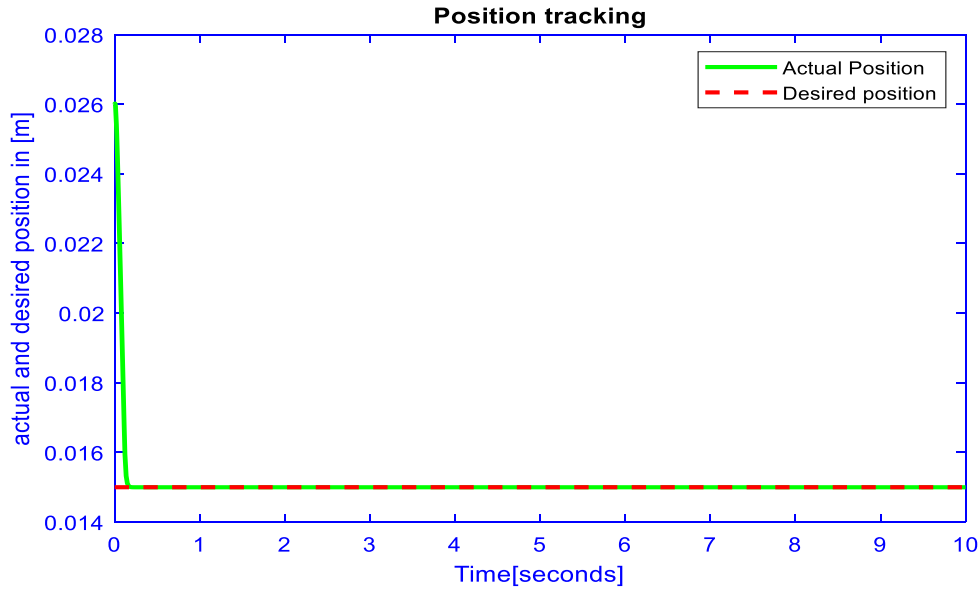


Figure 5. 5 Tracking desired position of 15mm starting from initial position of 26mm

The Fig.5.5 and 5.6 shows the tracking performance of the controller for maglev system. From the tracking error, the error dynamics approach to zero starting from the given initial point of 0.026m. Maglev system achieves the tracking perfectly after error will approach to zero. Similarly, from Fig.5.7. we can see the control signal required to achieve the goals form starting initial point and magnitude voltage required to achieve tracking nearly 5V for maglev for given position using SMC controller.

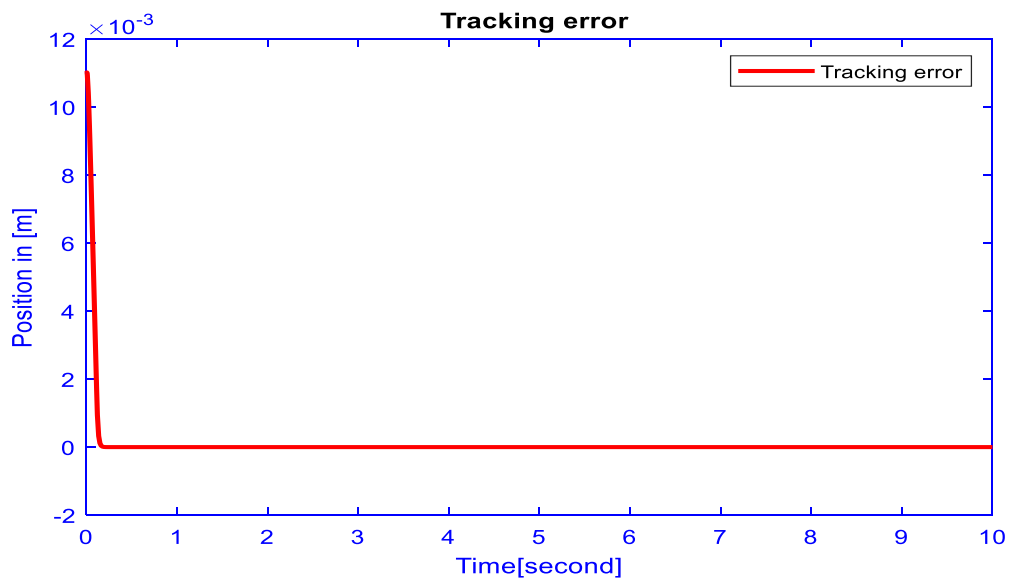


Figure 5. 6 Tracking error

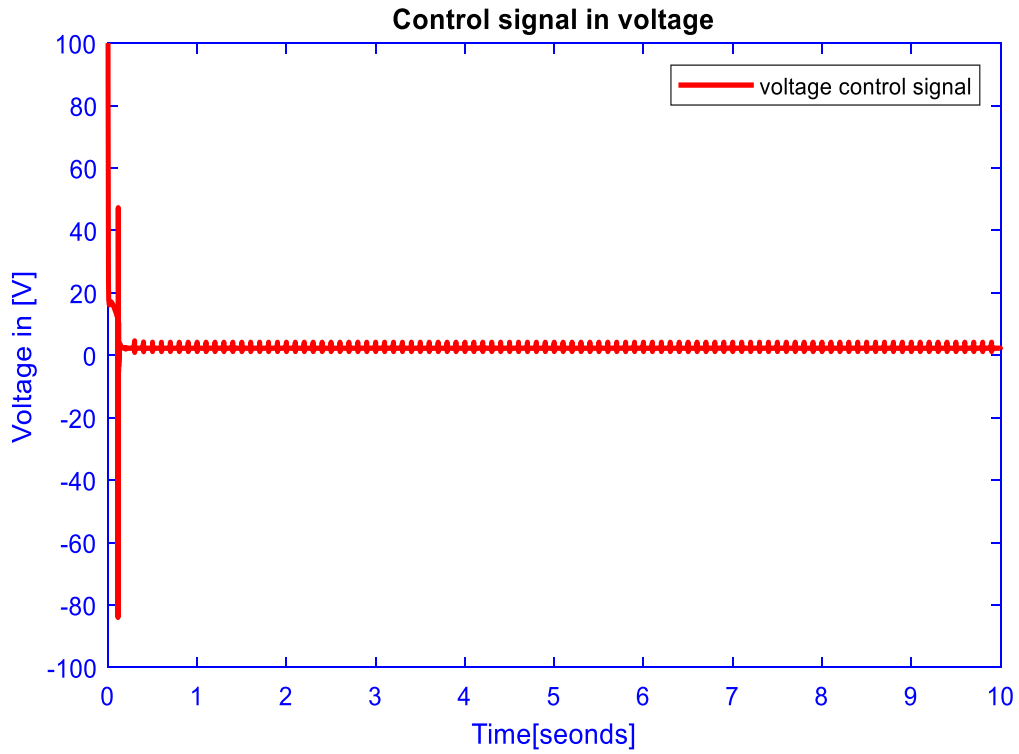


Figure 5. 7 Voltage control signal

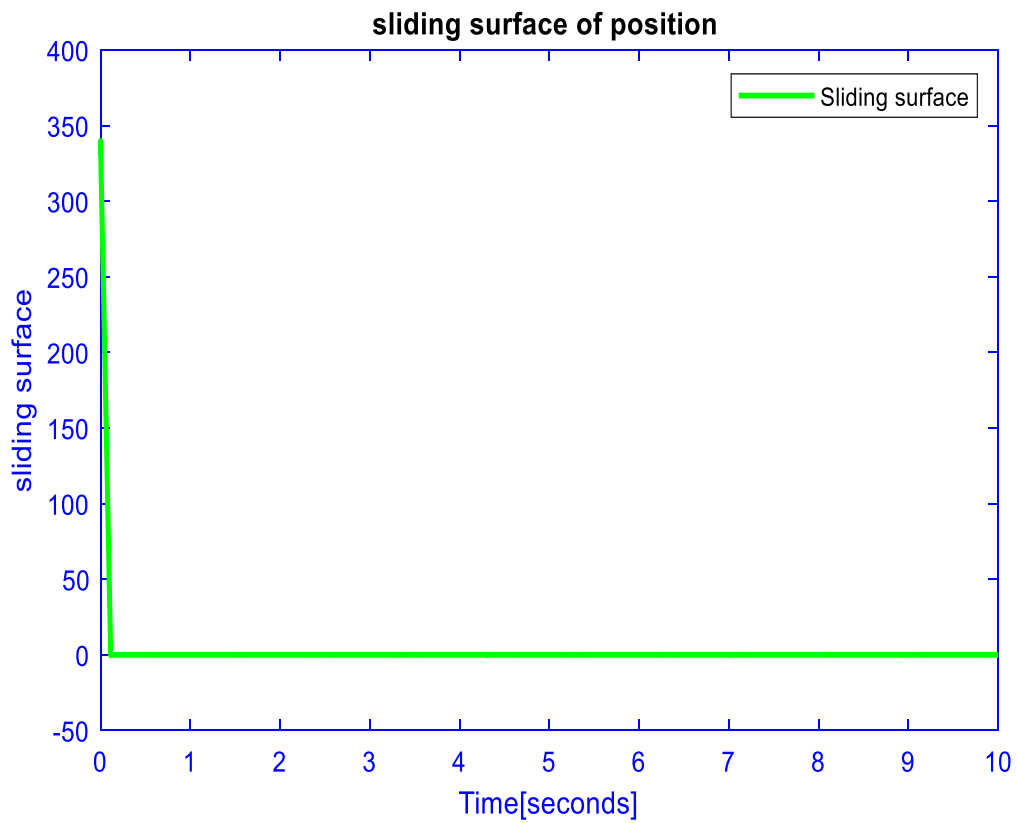


Figure 5. 8 Sliding surface

The sliding surfaces of PSO-SMC using arc tangential improved and it is so smooth for given optimal gain of SMC as shown in Fig. 5.8., since arc tangential techniques one of method to reduce the chattering methods.

5.1.3. Ball position tracking under matching model uncertainty

In this section maglev system is subject to 20% deviation $\Delta a_1, \Delta a_2, \Delta a_3$ and Δb from of nominal value of a_1, a_2, a_3 and b due to system parameters and loading mass uncertainty. Magnitude deviation variables used in this simulation are $\Delta a_1 = 0.2, \Delta a_2 = 0.2, \Delta a_3 = 0.2$ and $\Delta b = 0.2$. From the simulation result below Fig.5.6 and 5.7, we get that main advantage of the slide mode controller which is robustness to disturbances, as shown in tracking error, matching model uncertainty disturbance of 20% deviation due to system parameter system and loading mass applied to the maglev system. But, the proposed controller totally insensitive to model uncertainty. Moreover, the control signal also totally free from chattering problems and the amount of voltage control signal required to error zero using PSO-SMC method as indicated in Fig. 5.9.

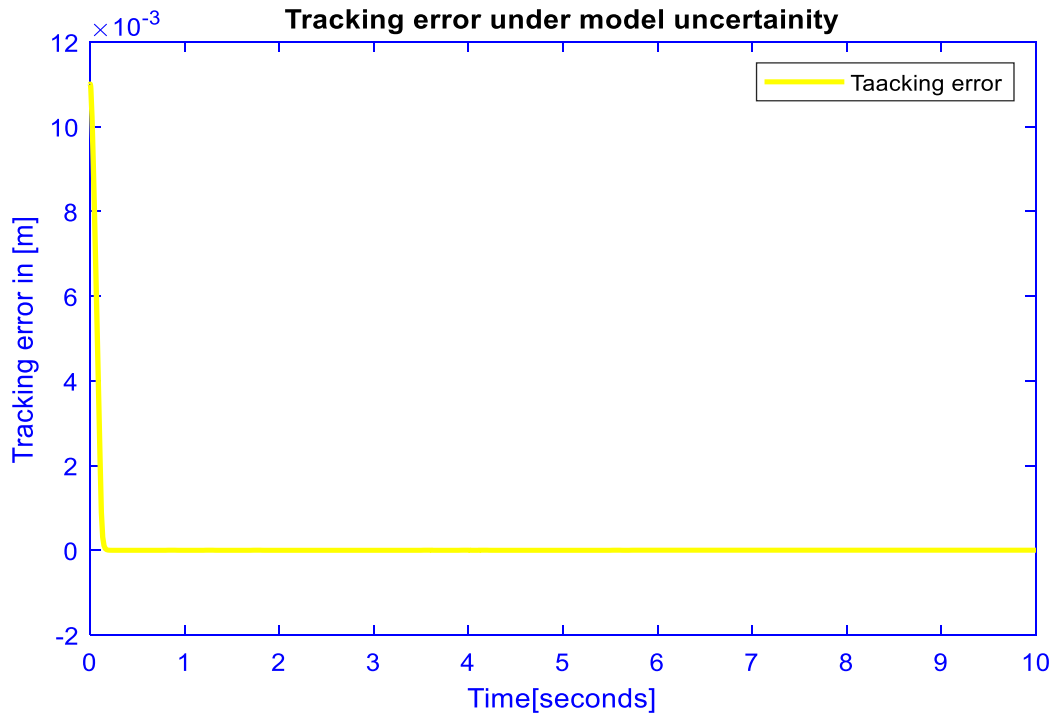


Figure 5. 9 Tracking error under matching model uncertainty

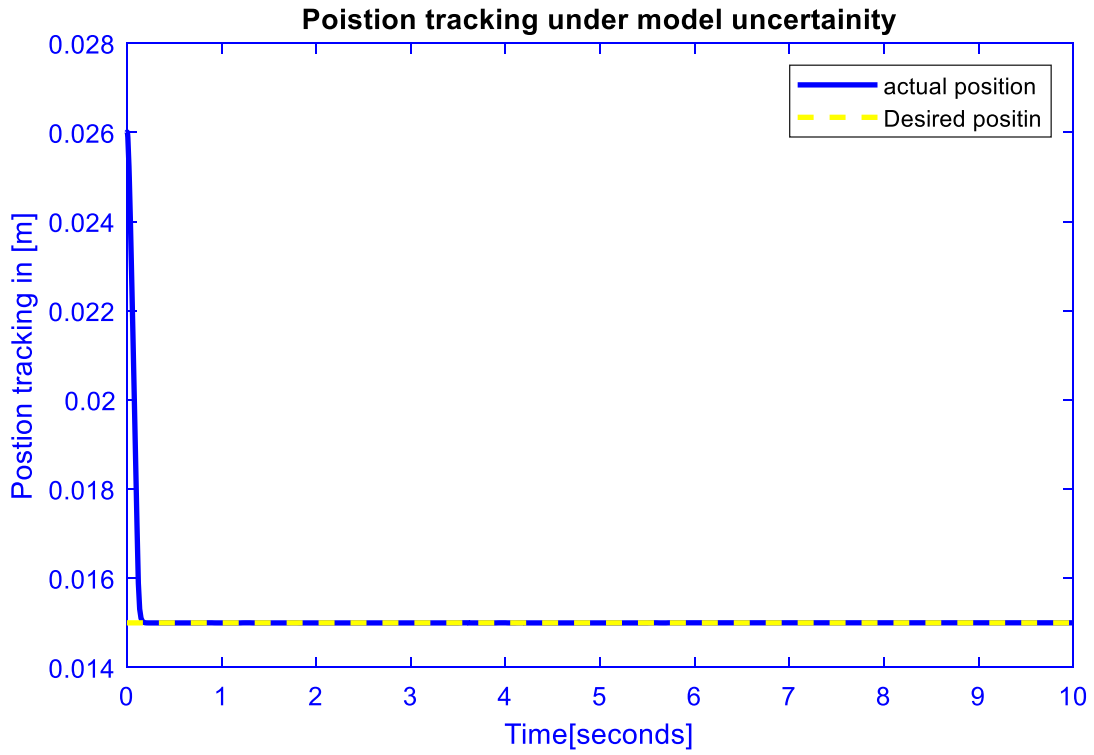


Figure 5. 10 Tracking of desired ball position under model uncertainty

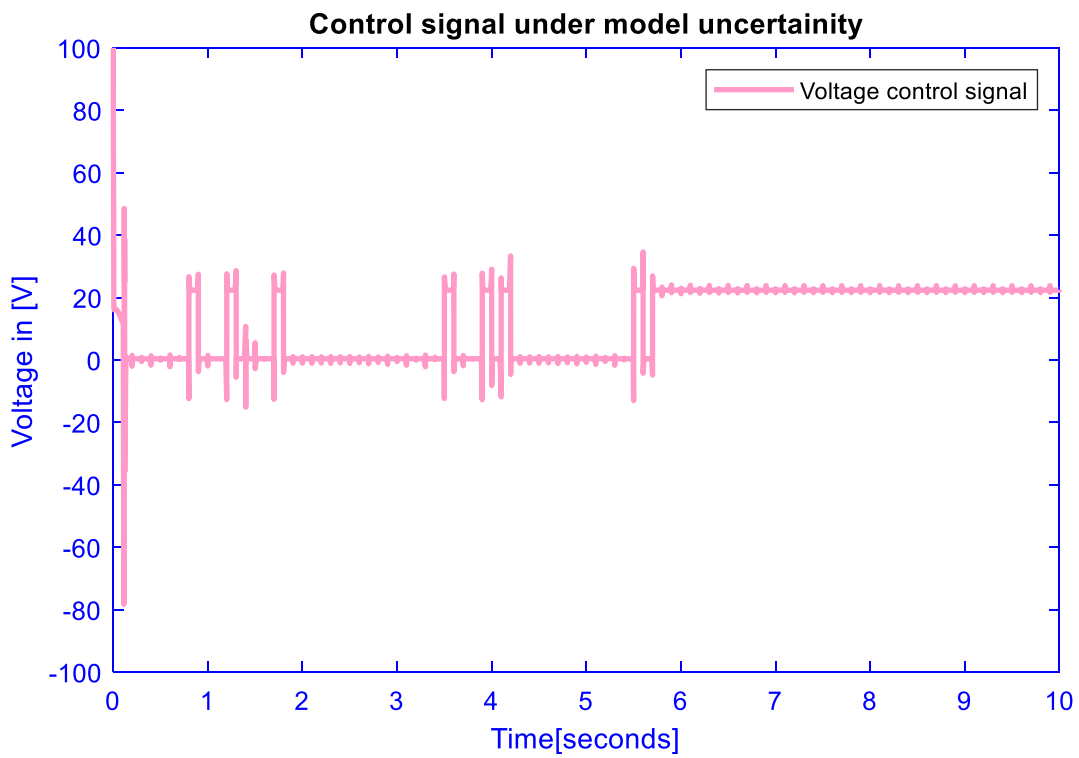


Figure 5. 11 Voltage control signal under model uncertainty

In case of Fig 5.12, we can see that, the sliding surface is oscillation free in another word chattering free throughout the 10 seconds.

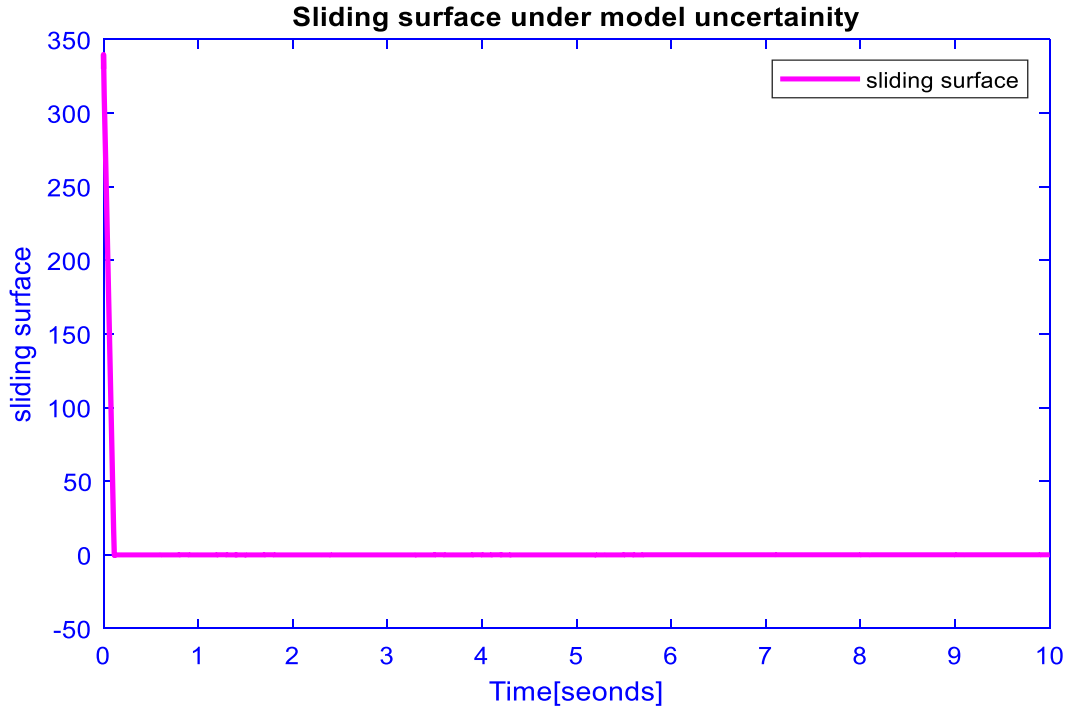


Figure 5. 12 Sliding surface under model uncertainty

5.1.4. Control System Under Matching Model Uncertainty and Random External Disturbance

The following matching model uncertainty (internal disturbance) and external random disturbance is applied to dynamics of maglev systems as follows.

Table 5. 4 Random external and internal (model uncertainty) disturbance value

Dynamics	Matching model uncertainty disturbance	External random Gaussian disturbance
Maglev system	$\Delta a_1 = 0.2, \Delta a_2 = 0.2, \Delta a_3 = 0.2$ and $\Delta b = 0.2$ which is 20% of deviations	Variance =10, seed =0 and mean=0

The magnitude of external random Gaussian disturbances applied to controller shown in Fig. 5.13, with variance of 10, seed and mean of zero.

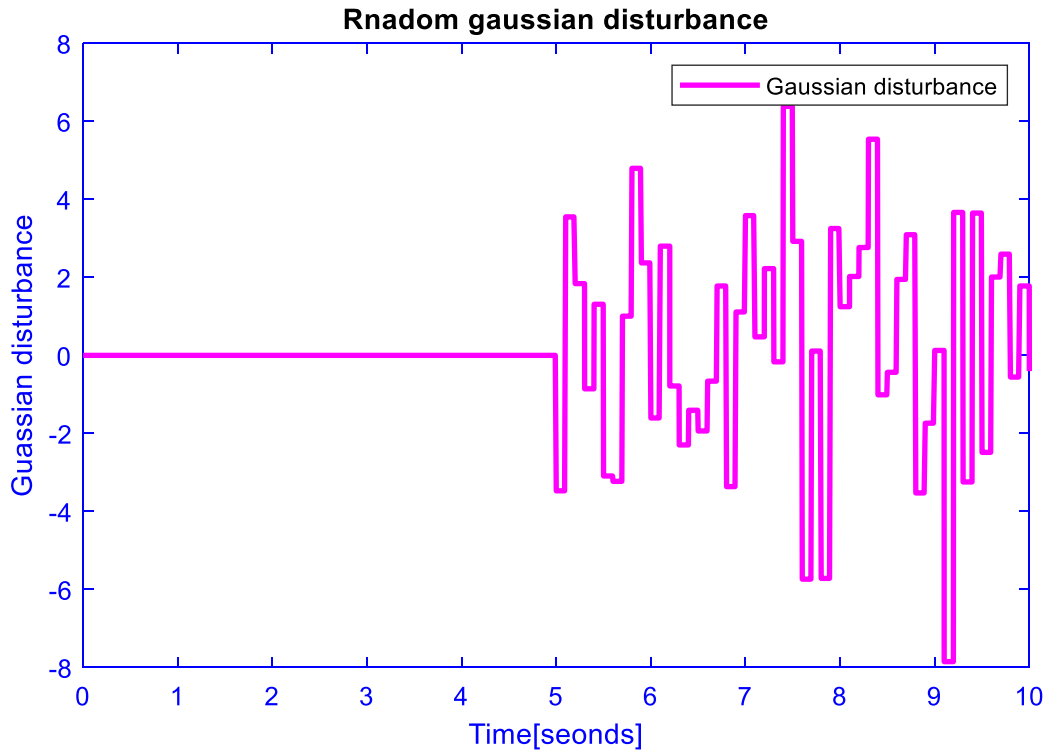


Figure 5. 13 Random Gaussian disturbance

From the simulation result below Fig.5.14 and 15, random Gaussian disturbance applied to the control output $t=5\text{sec}$ for maglev system. But, the proposed controller totally insensitive to model uncertainty and in case of external Gaussian disturbance applied to the magnetic levitation system after 5 seconds as shown above tracking error will never have affected by application of Gaussian disturbance. The PSO-SMC controller reacts accordingly and brings error back to zero shown Fig. 13 above. In general, the error starting from initial point of 0.026m approaches to zero and completely robust to zero external disturbances.

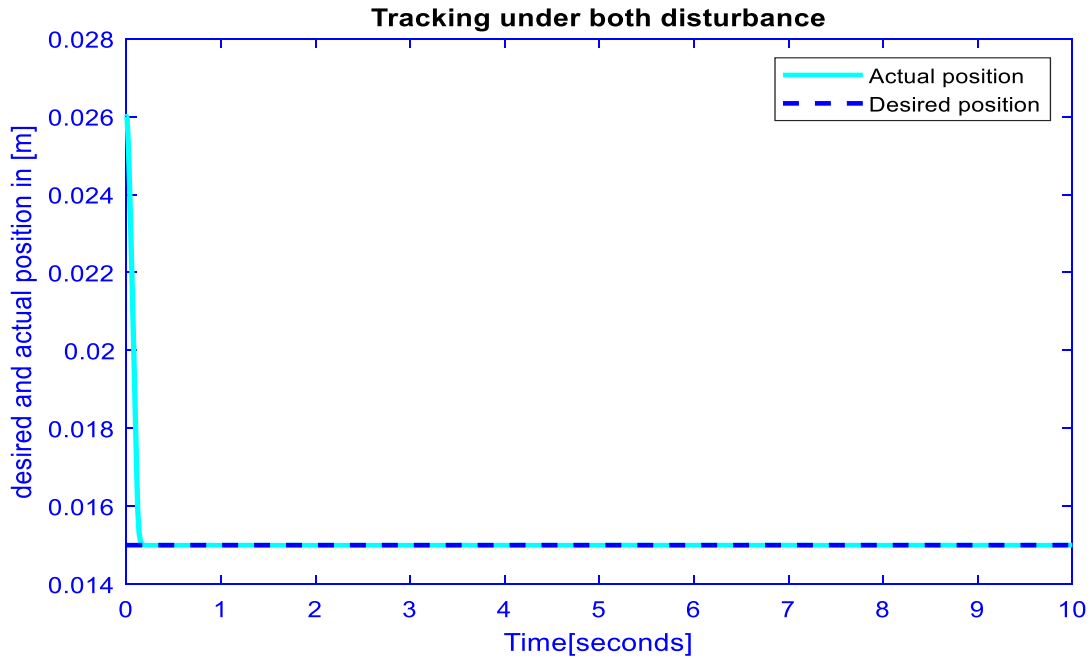


Figure 5. 14 Tracking error under both model uncertainty and external disturbances

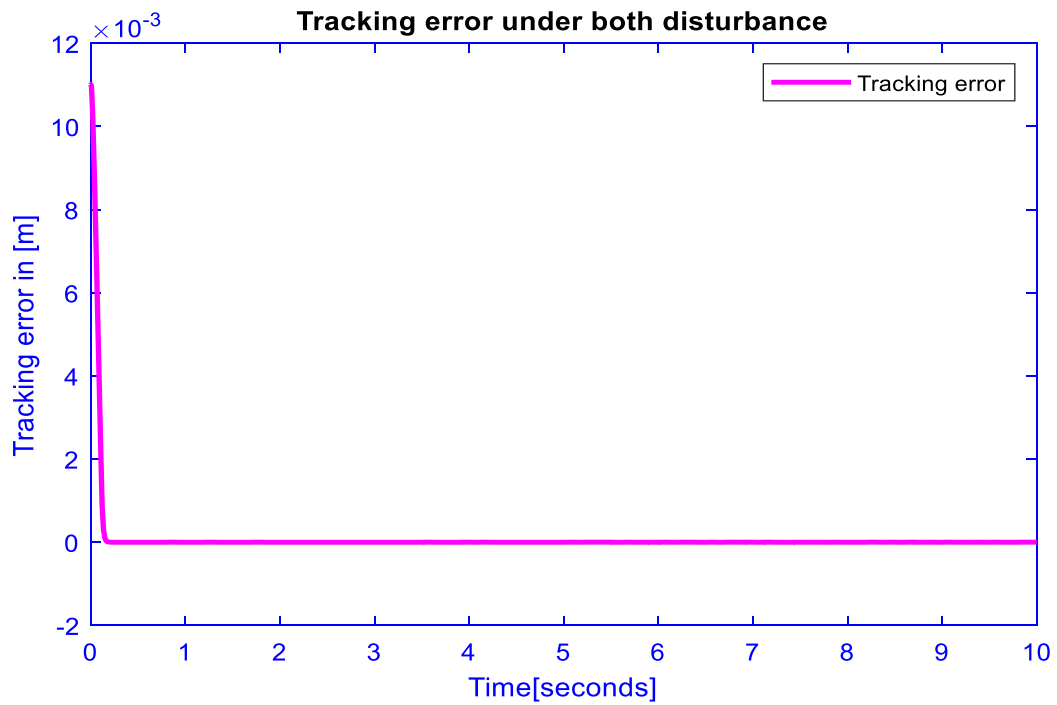


Figure 5. 15 Tracking error under both model uncertainty and external disturbances

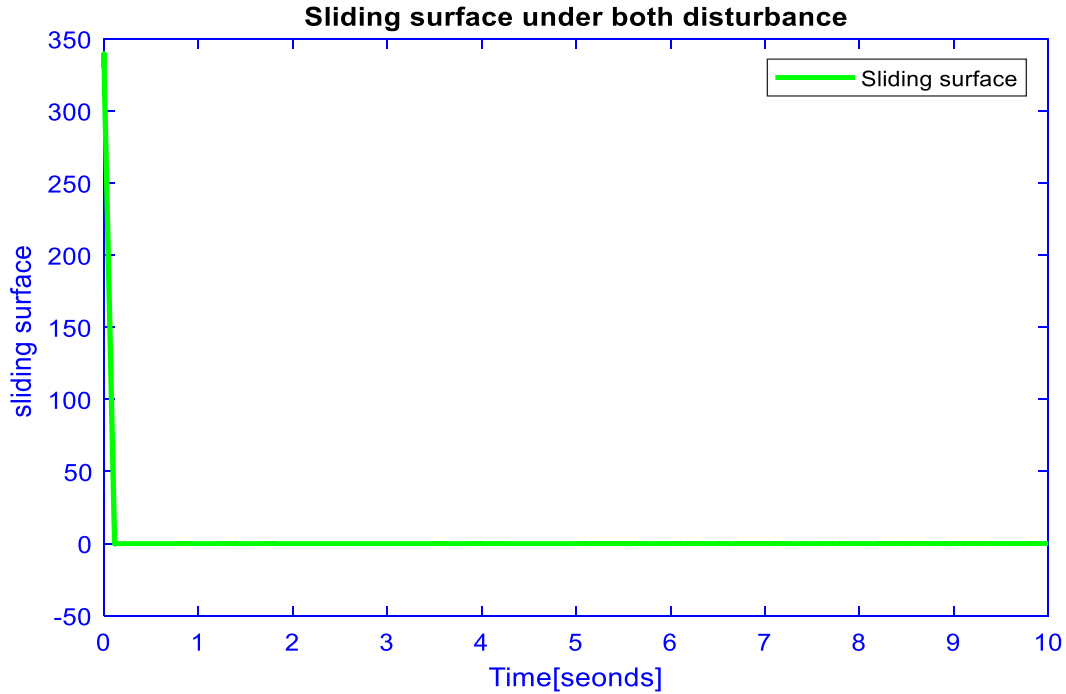


Figure 5. 16 Sliding surfaces under model uncertainty and external disturbance

Practically, chattering phenomena should have avoided because may cause damage to actuators, electronic devices due to high frequency oscillation and therefore, sliding surface must be smooth as much as possible in order to physical implement controller in real world applications. Different technique is proposed to reduce chattering phenomena in sliding mode controller such as saturation function, quasi-SMC or relay function, arc tangential function. As shown from the Fig.5.16, it's clear that sliding surface the of controller free from high frequency oscillation due to switching function. In this project arc tangential continuous function used in place of sign function, which is very good results in terms of fast convergence and to remove chattering effect. Moreover, proposed controller totally robust with respect matching model uncertainty i.e., internal disturbance and external disturbance.

From Fig 5.16, PSO-SMC will reject both disturbances after random disturbance is applied at 5secs maglev system. Magnitude of voltage required to achieve tracking and how to reject both disturbances indicated in the figure shown above. Since, applied signal is random Gaussian disturbance with vary in magnitude by variance of 10, so that controller should switch accordingly in order to avoid error to be suddenly increase from zero. Due to applied external random Gaussian disturbances magnitude of control signal increases and decreases from the previous case by 5V to 6V as shown, which nearly between 25V to 40V required to achieve main goals.

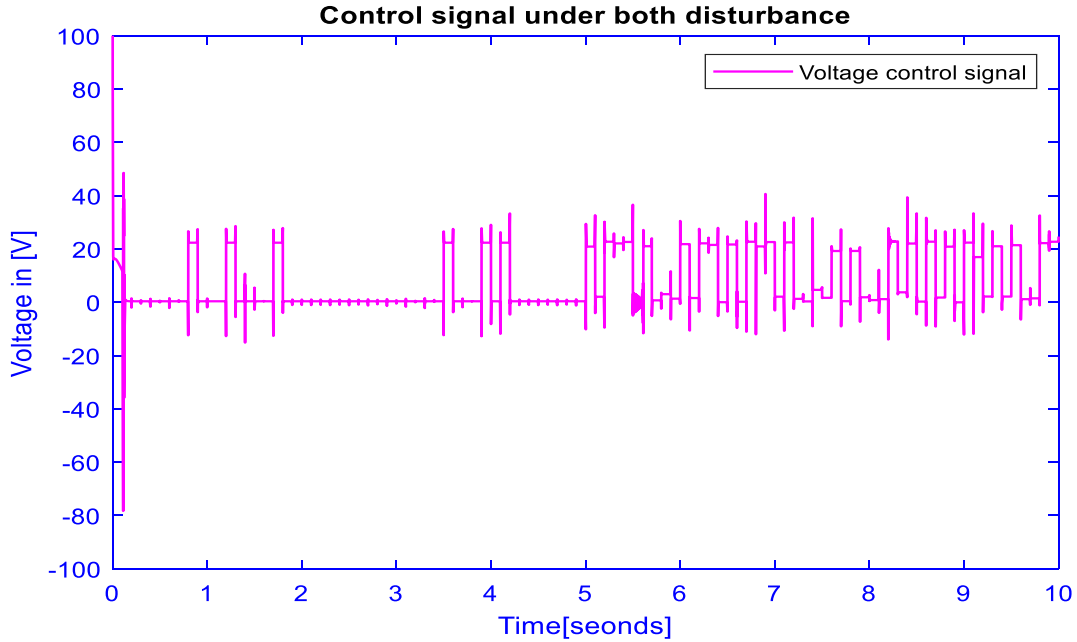


Figure 5. 17 Control signal under model uncertainty and external disturbances

Fig. 5.17, in case indicates trajectory tracking of reference trajectory of $x_2(t) = (15 + 15 \sin(0.2 * t))\text{mm}$ controller performances. The minimum integral square error obtained under this reference signal is 0.0052917. The magnitude parameters of controller are which similar to the previous value.

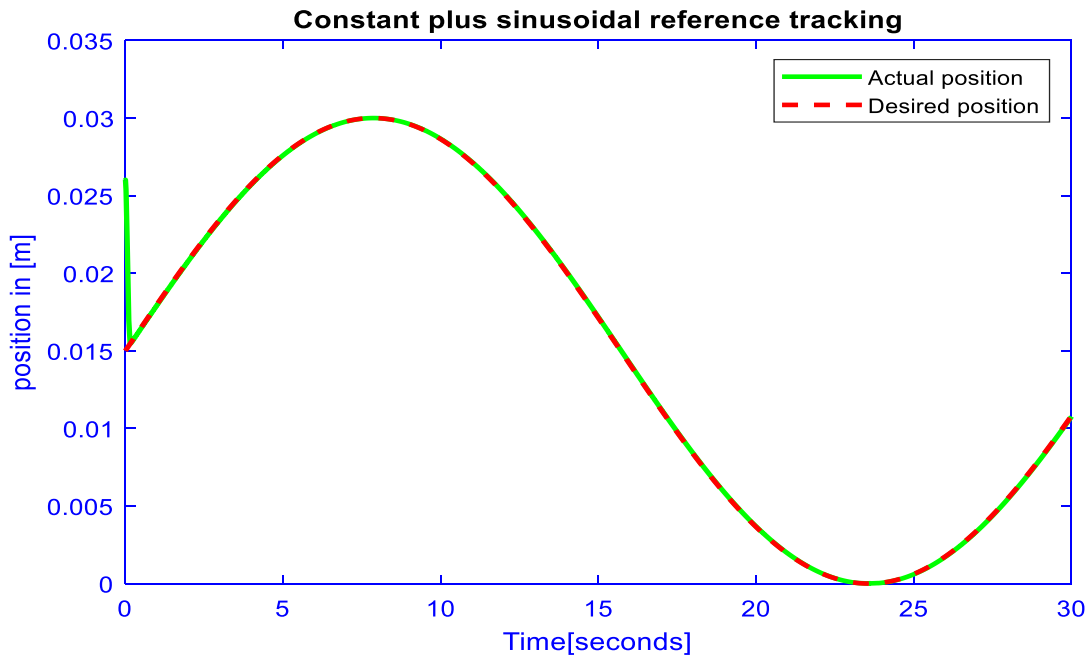


Figure 5. 18 Constant plus Sinusoidal reference position

CHAPTER SIX

CONCLUSIONS AND FUTURE RECOMENDATION

5.2. Conclusion

In this thesis, the dynamics of a maglev system are derived and a control law for ball position of maglev system is proposed. The position control for the magnetic levitation system is designed to achieve the desired reference position such that the error between the actual and desired position to comes zero using optimal sliding mode control system. Optimization base on adaptive particle swarm techniques for sliding mode control system to yield minima error between the actual and desired position using ISE, ITSE, IAE and ITAE fitness function. Effectiveness the proposed control system is tested against internal disturbance due system parameters and loading mass for matching model uncertainty. Moreover, random Gaussian external disturbance is applied to the system for robustness of the proposed control system at output of control signals. Therefore, by choosing appropriate value sliding mode control and arc tangential continuous function in place of signum function better ball position tracking is achieved and the effect of chattering phenomenon is reduced. Minimum error between desired and actual ball position was recorded using ITSE with magnitude of 0.00055 and maximum error occurred using IAE with magnitude of 0.683. In generally, designed controller is robust to both model uncertainty and external disturbance. Finally, the simulation results verify the performance of this proposed control, and can achieve good convergence of the position tracking errors.

5.3. Recommendation for the future work

The following suggestions are recommended for future works

- Nonlinear finite time convergence rather than asymptotical convergence such as terminal sliding mode control system with adaptive particle swarm optimization.
- Black box modelling based on system identification controller designed might result better due difficult in unstable open loop dynamic model of maglev system based on differential equation.
- Dynamic sliding mode with adaptive particle swarm which very robust with respect to static sliding mode control system.

Furthermore, hardware implementation of the designed controller practically should be perused using digital control system on microcontroller would be a very interesting extension of this work.

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APPENDIXES

```
%% MATLAB code PSO-SMC for Maglev system

clc; clear all; close all;
%Model parameters
tf=10;
% pso constant parameters
c=2; particles=20; iteration=200; var=1; var1=1; var2=1;
% Search space
a=0;
b=30000;
a1=0;
b1=30000;
a2=475;
b2=500;
wmax=0.9;
wmin=0.1;
%Initialization position and velocity of each decision variables
for m=1: particles
    for n=1: var
        v(m,n)=0;
        x(m,n)=a+rand*(b-a);
        xp(m,n)=x(m,n);
    end
    k1=x(m,1);
    for n=1: var1
        v1(m,n)=0;
        x1(m,n)=a1+rand*(b1-a1);
        xp1(m,n)=x1(m,n);
    end
    k2=x1(m,1);
    for n=1: var2
        v2(m,n)=0;
        x2(m,n)=a2+rand*(b2-a2);
        xp2(m,n)=x2(m,n);
    end
    k5=x2(m,1);
    %Run Simulink
    sim ('magnetic levitation')
    ISE(m)=sum(se_sim);
end
% Best global value
[best_performance, location] =min (ISE);
fg=Best_performance;
xg (1) =x(location,1);
xg1(1) =x1(location,1);
```

```

xg2(1) =x2(location,1);

                %Main loop
for i=1: iteration
    w=wmax-((wmax-wmin)/iteration) *i;
    for m=1: particles
        disp(['iteration' num2str(i) ': best cost=' num2str(fg)]);
        for n=1: var
            v(m,n)=w*v(m,n)+c*rand*(xp(m,n)-x(m,n))+c*rand*(xg(n)-x(m,n));
            x(m,n)=x(m,n)+v(m,n);
        end
        for n=1: var
            if x(m,n)<a;x(m,n)=a; end
            if x(m,n)>b;x(m,n)=b; end
        end
        k1=x(m,1);
        for n=1: var1
            v1(m, n) =w*v1(m,n)+c*rand*(xp1(m,n)-x1(m,n))+c*rand*(xg1(n)-x1(m,n));
            x1(m,n)=x1(m,n)+v1(m,n);
        end
        for n=1: var1
            if x1(m,n)<a1;x1(m,n)=a1;end
            if x1(m,n)>b1;x1(m,n)=b1; end
        end
        k2=x1(m,1);
        for n=1: var2
            v2(m, n) =w*v2(m,n)+c*rand*(xp2(m,n)-x2(m,n))+c*rand*(xg2(n)-x2(m,n));
            x2(m,n)=x2(m,n)+v2(m,n);
        end
        for n=1: var2
            if x2(m,n)<a2;x2(m,n)=a2;end
            if x2(m,n)>b2;x2(m,n)=b2; end
        end
        k5=x2(m,1);
        %Run Simulink
        sim ('magnetic levitation')
        % New personal best value
        ISE1(m)=sum(se_sim);
        if ISE1(m)<ISE(m)
            ISE(m)=ISE1(m);
            xp(m,1) =x(m,1);
            xp1(m,1) =x1(m,1);
            xp2(m,1) =x2(m,1);
        end
        end
        end
        %Global best value

```

```
[B_fg,location]=min(ISE);  
if B_fg<fg  
    fg=B_fg;  
    xg (1) =xp(location,1);  
    xg1(1) =xp1(location,1);  
    xg2(1) =xp2(location,1);  
end  
  
end  
    k1=xg (1);  
    k2=xg1(1);  
    k5=xg2(1);
```