MATHEMATICAL MODELING OF ASPECT OF CROCODILE RANCHING WITH NUMERICAL ANALYSIS APPROACH

Hirut Gebremedhin Yimer

A Thesis Submitted to
The Department of Applied Mathematics
School of Applied Natural Science

Presented in Partial Fulfillment of the Requirement for the Degree of Master's in Applied Mathematics (Numerical Analysis)

Office of Graduate Studies
Adama Science and Technology University

June 2024
Adama, Ethiopia

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Hirut Gebremedhin Yimer
Supervisors: Prof. Anne Kværnø and Dr. Mesfin Mekuria Woldaregay

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DECLARATION

I here by declare that this Masters Thesis entitled ***"Mathematical Modeling of aspect of crocodile ranching with numerical analysis approach"*** is my original work. That is, it has not been submitted for the award of any academic degree, diploma, or certificate in any other university. All sources of materials that are used for this thesis have been duly acknowledged through citation

______________________	________________	______________
Name of student	Signature	Date

RECOMMENDATION

We, the advisors of this thesis, hereby certify that we have read and revised version of the thesis entitled ***"Mathematical Modeling of aspect of crocodile ranching with numerical analysis approach"*** prepared under our guidance by **Hirut Gebremedhin Yimer** submitted in partial fulfillment of the requirements for the degree of Master's of Science in Applied Mathematics. Therefore, we recommend the submission of revised version of the thesis to the department following the applicable procedure.

Anne Kværnø	Anne Kværnø	July 7, 2024
Major Advisor	Signature	Date
Co-advisor	Signature	Date

APPROVAL PAGE

We, the advisors of the thesis entitled ***"Mathematical Modeling of aspect of crocodile ranching with numerical analysis approach"*** and developed by **Hirut Gebremedhin Yimer**, hereby certify that the recommendation and suggestions made by the board of examiners are appropriately incorporated into the final version of the thesis.

Anne Kværnø	Anne Kværnø	July 1, 2024
Major Advisor	Signature	Date
Co-advisor	Signature	Date

We, the undersigned, members of the Board of Examiners of the thesis by **Hirut Gebremedhin Yimer** have read and evaluated the thesis entitled ***"Mathematical Modeling of aspect of crocodile ranching with numerical analysis approach"*** and examined the candidate during open defense. This is, therefore, to certify that the thesis is accepted for partial fulfillment of the requirement of the degree of Master of Science in Applied Mathematics.

Chairperson	Signature	Date
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External Examiner	Signature	Date
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School Dean	Signature	Date
Office of Postgraduate Studies Dean	Signature	Date

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ABSTRACT

Crocodiles are aquatic reptile which includes 23 species in the world. This thesis investigated the growth patterns of crocodiles in Arba Minch Crocodile Ranch, employing three renowned growth models: Von Bertalanffy, Logistic and Von Bertalanffy-Putter. The study aims to identify the best fit model by comparing their performance using the least square method in python. The Bertalanffy Putter model was further modified to include linear terms, enhancing its adaptability to the crocodile growth data. The performance of each model was evaluated by calculating the sum of squared differences.The results indicate that the modified Bertalanffy model provided the most accurate representation of the growth of Nile crocodiles. This model accurately captures the growth patterns. To further validate the models, they were applied to a separate data set from the literature with long term growth data. With some simulations we have seen that increasing the amount of food than usual decreases the amount of time to reach the desired length to slaughter the crocodiles.

Keywords: Crocodile Growth, Von Bertalanffy Model, Logistic Model, Bertalanffy-Putter Model, Modified Bertalanffy-Putter Model, Least Squares Method, Python, Sum of Squared Differences.

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LIST OF ACRONYMS

AMCR	Arba Minch Crocodile Ranch
NWCO	National Wildlife Conservation Organization
PLC	Private Limited Company
BP	Bertalanffy Putter
TL	Total length

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CHAPTER ONE
INTRODUCTION

1.1 Background of the Study

Crocodiles are large aquatic reptiles that live in tropical areas around the world, including Australia, Africa, the Americas, and Asia. The dwarf crocodile is the smallest of the 23 species, reaching less than 2 meters in length and weighing around 7 kilograms. The saltwater crocodile is the largest and which has length 6.17 meters and weighs up to 907 kilograms. As cold-blooded creatures, crocodiles rely on the sun's warmth, but they also use mud as a natural sunscreen to prevent dehydration when it gets too hot. Despite their remarkable characteristics, crocodiles have unfortunately become a valuable commodity, sought after for their meat and skin (Viva.org.uk, 2024).

Crocodiles are carnivores and eat birds, fish, crustaceans, mammals, and frogs. Big species such as the Nile Crocodile can bring down animals as big as buffaloes and zebras. As ambush hunters, they wait for their prey to come close and then lunge forward to attack. They clamp down their jaws on the prey, crush it and then swallow it whole. Crocodiles cannot chew their food and so swallow small stones to help digest it by grinding the contents of their stomachs. They have a very slow metabolism and can survive for months without food (Tyowua & Uloko, 2019).

Crocodiles live on average between 50 and 75 years, depending on the species. The oldest crocodile in captivity reached the age of around 140 years. He was called Mr. Freshi, a freshwater crocodile, and lived in an Australian zoo (Sangha, Stoeckl, Crossman, & Costanza, 2019). We can see different life stages of Nile crocodiles from Figures (1.1,1.2,1.3,1.4).

Crocodiles are bred to be exploited for their skin and meat in three ways: They are bred captive on factory farms; they are 'ranched', where eggs, hatchlings, or juveniles are taken from the wild and bred on farms; or they are 'wild harvested'. The sale of their skins is more profitable than their meat.

Around 1.33 million crocodiles were killed each year worldwide from 2007 to 2010 and over 1.5 million in subsequent years. Their meat is a by-product of the production of their expensive skins. Crocodiles are legally exported from 30 different countries around the world. The US traded 481,000 skins of Alligator mississippiensis in 2013, and similar numbers in other years, which makes them another big producer of crocodile (Viva.org.uk, 2024).

Australia accounts for just over one percent of the world's production of crocodile meat. Trade from African countries is increasing by 22 percent per year. Zimbabwe is the biggest producer of crocodile meat in Africa, while Kenya exports 85 percent of its production to China, Hong Kong, and Taiwan. Today, there are 13 crocodile farms in Australia alone and the majority of skins and meat are exported. The farmer uses fallen chickens and pigs and other animal waste from intensive factory farms as food for the animals, who are slaughtered and sold commercially (Saalfeld et al., 2016).

Thailand is the world leader in crocodile farming, with around one million animals on 20 mega farms and around 1,000 farms in total. The largest farm has around 150,000 animals. Every morning, the crocodiles' eggs are collected and placed in a specially built incubator room. After 45 days the eggs start to hatch, with the big farms hatching around 20,000 baby crocodiles each year (Adan, 2000).

Crocodiles reach slaughter weight when they are around 1.8 meters to 2.4 meters, between two and three years of age (Sangha et al., 2019). Around 1.33 million crocodiles were killed each year worldwide from 2007 to 2010 and over 1.5 million in subsequent years. Their meat is a by-product of producing their expensive skins. China has the biggest demand for crocodile meat, with some parts of the animals' bodies being used in pharmaceuticals and traditional Chinese medicine. Crocodiles are legally exported from 30 different countries around the world. The US traded 481,000 skins of Alligator mississippiensis in 2013, and similar numbers in other years, which makes them another big producer of crocodile meat too. Thailand exports most of its vast amounts of crocodile meat to China. The most lucrative part of all crocodile farming is their skins and just one crocodile skin can be sold for roughly USD 300-400 (Viva.org.uk, 2024).

Crocodiles play important role in preserving the structure and function of ecosystems. The largest *Crocodylus niloticus* discovered in Lake Chamo, a Rift valley lake in southern Ethiopia was some over 5.5 meters and they can reach up to 6 meters (Whitaker & Whitaker, 2008). Crocodile densities shows a strong relation ship with human population growth and it is more likely that the status of crocodiles throughout Africa is influenced by human population growth and growth patterns (Aust, 2009).

The Nile Crocodile is the most employable crocodilian species, its skin is highly regarded a classical material (Fergusson, 2010). Crocodile leather products are highly

valued for their unique beauty, longevity, and a typical qualities (Chala et al., 2020). In their natural habitat, Nile crocodiles experience a mortality rate over 90% in the first three years of life. Conversely, in hatcheries, this rate is greatly reduced to under 30%. Once these crocodiles grow to a length of approximately one meter, they have nearly no remaining natural enemies, with the exception of human beings (Dembner, 1990).
Crocodile ranching has emerged as a significant industry worldwide, contributing to economic growth, conservation efforts, and the sustainable utilization of crocodile populations. However, the management of crocodile ranches presents various challenges, including the need to optimize breeding practices, feeding schedules, and population dynamics to ensure the long-term viability and profitability of these operations. In the

Figure 1.1: Crocodile baby egg (source www.google.com)

context of crocodile ranching, the application of mathematical models can provide insight into the growth and reproduction patterns of crocodiles, the impact of environmental factors on their development, and the effectiveness of different management strategies in enhancing productivity and sustainability.

Holding crocodiles in captivity for breeding purposes is not a new idea. Crocodiles have been bred in farms since early 20th century. The majority of these farms were tourist attractions with wild caught alligators or crocodiles under captivity (Masser, 1993; Stickney, 2000). In Africa, many ranches prefer Nile Crocodiles for farming. Saltwater crocodiles are preferred in Australia. caiman also take place in South America(Tisdell & Swarna Nantha, 2005; Revol, 1995). Commercial crocodile farming began in South Africa in the 1970s and has become an international enterprise (Chabreck & Joanen, 1979). However feeding crocodiles is a greatest challenge in the day-to-day running of a crocodile farm, particularly in developing nations (Wallace, 2006). Crocodile farming is a highly complicated industry that requires careful management of various factors to maximize skin production. Factors such as feeding schedules, temperature control, breeding strategies, and growth rates all play a critical role in determining the overall productivity of a crocodile farm. To effectively optimize these variables and their interactions over time, the use of dynamic optimization models can be highly beneficial.

Nile crocodile is the only crocodile species in Ethiopia and distributed in lakes Abaya, Chamo, Beseka, Chew Bahir, and different river basins of Ethiopia such as Abay, Baro-Akobo, Wabishebele, Awash and Omo-Gibe (Abate, 2022). *Crocodylus niloticus* is widely distributed in the southern half of Ethiopia. From all, Lake Chamo has the largest and most significant population of crocodiles (Shirely et al., 2014). In the 1960s, despite intense hunting, Nile crocodile populations have managed to endure in more inaccessible rivers and lakes. On lakes Abaya and Chamo in the Rift Valley, there are concentrations of crocodiles unrivaled in Africa.

The concept of captive rearing of crocodiles in Ethiopia is not new; National Wildlife Conservation Organization(NWCO) has followed the progress of crocodile management schemes developed in other parts of the world. A ranching program for *Crocodylus niloticus* was implemented in Ethiopia in 1985 when the government created Arba Minch Crocodile Ranch(AMCR) (Shirely et al., 2014) (Bolton, 1989). The Nile crocodile is not listed as a protected species in Ethiopia and, because of this, consumptive use programs are permitted under Ethiopian legislation. The species is currently subject to both consumptive (e.g., ranching and trophy hunting) and non-consumptive (e.g., tourism) uses, as well as implicated in human-wildlife conflict (Shirely et al., 2014).

Despite the potential benefits of mathematical modeling in crocodile ranching, there remains a gap in the literature regarding comprehensive models tailored specifically for this industry. Existing studies have focused on general population dynamics or broad ecological interactions, rather than modeling their growth mathematically and trying to control specific challenges faced by crocodile ranchers in decreasing the time it takes to grow to a certain length in a least amount of time.
A crocodile farm is a closed-cycle captive breeding establishment that is managed so that crocodiles have artificial housing, veterinary care, artificially supplied food, and protection from predators. This culture application is mostly intended for producing skins, meat, oil, and claws (Stickney, 2000).

Modeling involves the creation of a model, which serves as a representation of a particular system and its functionality. The model is a simplified version of the system, designed to assist in predicting the impact of changes made to the system. Ideally, a

Figure 1.2: Photo of 7 years old crocodiles from Arba Minch crocodile ranch.

Figure 1.3: Photo of 8 years old crocodiles from Arba Minch crocodile ranch.

model should closely resemble the actual system and encompass its key characteristics, while also being comprehensible and not overly complicated. A successful model strikes a balance between realism and simplicity, enabling effective experimentation and analysis (Maria, 1997).

Optimal control is a mathematical technique derived from the calculus of variations. There are several different methods for calculating the optimal control for a specific mathematical model. For example, the Pontryagin maximum principle allows the calculation of the optimal control for an ordinary differential equation model system with a given constraint. Variations of the Pontryagin maximum principle have been derived

Figure 1.4: Crocodile(source www.google.com)

for other types of models including partial differential equations and difference equations (Kamien & Schwartz, 1991; Lenhart & Workman, 2007).

In this thesis, we have developed a dynamic optimization model for crocodile farming as a case study in Arba Minch Crocodile Ranch (AMCR) that takes into account the various factors influencing skin production. By utilizing this model, one can identify the most effective feeding schedules to enhance overall productivity. Through ongoing monitoring and adjustment of the model based on new data, we can continuously improve farming practices and ultimately increase skin production in crocodile farming operations like Arba Minch Crocodile Ranch. Some products of crocodiles skin can be seen in Figure(1.5).

1.2 Statement of the problem

Ethiopia has been exporting Nile crocodile *Crocodylus niloticus* since the early 1990s (Shirely et al., 2014). This study will provide a model to the growth of crocodiles and optimal control on their feeding. By utilizing this model one can identify the most effective feeding schedules, temperature control measures and breeding strategies to enhance overall productivity.

As we can see from Table 1.1, there is a huge difference between the amount Ethiopia earn and some other countries earn from crocodile ranching. If we can feed the crocodiles well we do not need to raise them until 7 or 8 years but slaughter them in 3 years old (Furstenburg, 2008). The goal of this study is to address the following questions

- how to develop a mathematical model that describes the growth of a crocodile?

Figure 1.5: Some of crocodile skin products (source www.google.com)

- what are the conditions to determine the existence and stability of equilibrium points?
- what is the advantage of modeling crocodile growth?
- what is the advantage of feeding with proper schedules and proper amount for crocodile growth (taking feeding as control input)?

Table 1.1: Some countries with the money they have earned from crocodile.

Country's name	Year	Amount earned in USD
Louisiana state in USA	2021	$80,201,270 (Rosales Chiessa, 2021)
South Africa	2005	$400,000 (Furstenburg, 2008)
South Africa	2012	$ 10 M (Mendal, 2012)
South Africa	2013	$ 6.75 M (Lindsey et al., 2013)
Zimbabwe	1996	$ 5 M (Dzoma et al., 2008)
Zimbabwe	2013	$ 26 M (Lindsey et al., 2013)
Zambia	2013	$ 3-4 M (Lindsey et al., 2013)
Mozambique	2013	$ 1.5 M (Lindsey et al., 2013)
Australia	2014	$ 64.44 M (Sangha et al., 2019)
Australia	2015	$ 33.67 M (Sangha et al., 2019)
Ethiopia	1989-2005	$ 96,125 (Shirely et al., 2014)

1.3 Objectives of the study

1.3.1 General objective

The general objective of this study is to develop and analyze a Mathematical Modeling of aspect of crocodile ranching with numerical analysis approach.

1.3.2 Specific objectives

- Formulate a mathematical model for crocodile growth.
- Perform analysis on the proposed mathematical model.
- Extend the developed mathematical model into optimal control.
- Fit the mathematical model with real data that we got from AMCR and solve it numerically.

1.4 Significance of the study

- insights researches are needed in this field.
- predicts the growth of crocodiles with the availability of resources and moderate temperature.
- guide crocodile farmers in AMCR or other crocodile farms on how to maximize skin production with available resources.
- motivates other researchers for further study and mathematical analysis.

1.4.1 Expected outcome

The expected outcome of this thesis is an optimal controlled mathematical model on optimizing the operation of a crocodile farm. This concrete model is restricted to certain aspects of the skin production, for example, the size of the crocodile with respect to feeding.

CHAPTER TWO
Literature Review

According to (Delene et al., 2020), the global export of Nile crocodile skins averaged 201,000 per year from 2007 to 2016, with a notable increase between 2009 and 2016. However, Arba Minch Crocodile Ranch in Ethiopia faces significant challenges, including nutritional deficiencies and health issues affecting the Nile crocodile population. The diseases affecting these animals can be categorized into two types: infectious diseases (bacterial, viral, fungal, protozoan, and parasitic) and non-infectious diseases (nutritional, toxic poisonings, and metabolic disorders). The ranch's major health concerns include nutritional bone diseases and skin lesions. They summarized the key diseases and management status of *C. niloticus* at Arba Minch ranches. To mitigate these issues they have suggested that , the Arba Minch Crocodile Ranch should exercise caution when introducing wild-hatched crocodiles, improve its husbandry practices to reduce disease incidence, and collaborate with experts and research groups.

Arega et al. (2022) identified major health problems and constraints affecting crocodile skin production Arba Minch Crocodile Ranch (AMCR). The study found that bacterial infections (34.3%) and non-infectious diseases like trauma (40%) and poisoning (37.1%) were prevalent. Contaminated feed and water, and poor sanitation were major contributors to health issues. Most interventions were carried out by non-veterinarians, and fungal diseases were the most common skin problem. The study recommended that developing a clear policy on crocodile management and implementing best practices to improve the industry.

Abate (2022) assessed threats to Nile crocodiles in Lake Chamo and risks to the local crocodile ranch. Data was collected through surveys, interviews, and observations from July 2021 to August 2022. The study found that human activities such as deforestation, farming, and pollution are major threats to Nile crocodiles in the lake. Meanwhile, the ranch faces risks including habitat disruption, lack of facilities, and market shortages. The majority of respondents agreed that the species is under threat due to human activities. To address this, awareness programs should be implemented, and the local government should establish a buffer zone around the lake to protect the crocodiles. Further research is needed to understand the population and distribution of Nile crocodiles in the lake.

Chabreck and Joanen (1979) utilized a capture-recapture method to analyze the growth rates of American alligators in Louisiana. They discovered a significant correlation between total length and both snout-vent length and weight. The comparison of small alligators revealed that the growth rates of males and females are similar until the animals reach a total length of 1.0 m, at which point the growth of females decreases significantly. Growth rates were observed to be highest during mid-summer, lower during spring and fall, and no growth occurred during winter (Oct-Mar). According to a mathematical model, males grow rapidly for approximately 20 years, reaching a projected total length of 4.20 m at age 80, with a decline in growth after reaching ≈ 3.50 m long. In contrast, the growth of females declines considerably after age 10, with individuals only reaching about 2.55 m in length at age 20. The maximum projected length for females was 2.73 m at age 45.

Jacobsen and Kushlan (1989) conducted a study on the growth patterns of the American alligator (*Alligator mississippiensis*) in the subtropical Florida Everglades, utilizing extensive mark-recapture data from over 2000 recaptures of both known-aged and unknown-aged animals. The growth of Everglades alligators was found to be best described by a power curve model. The nonasymptotic nature of their curve led to the rejection of the idea that alligator growth is determinate. The researchers proposed a model with piece-wise linear equations to better explain growth in the first year, indicating a period of halted growth during the first winter. Comparing growth models derived from various populations revealed that Everglades alligators exhibited slower growth compared to those in more temperate regions, contradicting the hypothesis that growth rates in subtropical Florida would be higher due to the longer growing season. This outcome was attributed to a combination of increased maintenance costs and a limited resource base in the Everglades.

Rootes et al. (1991) conducted a comparative investigation of the growth rates of American alligators (Alligator mississippiensis) in estuarine and palustrine wetlands in southwestern Louisiana. They found that alligators in estuarine wetlands, with salinity levels $\leq 5\%$, grew faster and reached sexual maturity earlier than those in palustrine wetlands, which are characterized by shallow, freshwater marsh vegetation. The slower growth rates in palustrine wetlands were attributed to lower prey density, as indicated by previous research. Additionally, males grew faster than females and reached sexual maturity at an earlier age in both habitats. This study highlighted the limitation of using total lengths as an index for population age structure, even when alligators are in the same geographic region.

Wilkinson and Rhodes (1997) conducted a study on growth rates of American alligators in South Carolina, aiming to determine the implications of slower growth rates in the northern portion of the alligators' range on life-history traits such as age and size of sexual maturity. Using capture-recapture data from 1972 to 1993, they found that males exhibited faster growth rates (20.2 cm/yr, 0-6 yr age; 7.22 cm/yr, 6 yr age to model asymptote) compared to females (18.0 cm/yr, 0-6 yr age; 6.34 cm/yr, 6 yr age to model asymptote), and reached a larger mean asymptotic size (M = 3.79 ± 0.08 SE m, F = 2.78 ± 0.04 m). The study also revealed that South Carolina alligators reached sexual maturity at an older age and larger body size than alligators in other regions. The researchers suggested that delayed breeding at a larger size in South Carolina may be more related to social dominance than to growth rates. They emphasized the importance of understanding age and size relations for effective management of alligators.

Tucker et al. (2007) analysed growth models for a population of Australian freshwater crocodiles *(Crocodylus johnstoni)*. Competing growth models were tested with two data sets: individuals of known-age, and growth interval data from capture-recapture records. A von Bertalanffy function provided the best empirical fit of several growth models. The estimated asymptotic lengths (snout–vent length of males = 125.3 cm; females = 97.4 cm) agreed well with average lengths of the ten largest males and females in the population. Sexual size demorphism in this species resulted from a combination of smaller mean length at maturity for females and a subsequent decline in female growth rate. Size dimorphism may result from individual trade-offs in age vs length at maturity as a consequence of sexual selection.

Garner et al. (2016) have studied that the growing interest in North Carolina in establishing a seasonal alligator harvest, but the potential effects of such a harvest on northern alligator populations were not well understood. To address that, they have created a model of alligator life history specific to females at the northern limit of their range. Their model allowed them to calculate the population growth rate (λ). They then conducted sensitivity and elasticity analyses to determine the impact of each vital rate on (λ). Additionally, they used a life-stage simulation analysis to account for variability in vital rates by incorporating hypothetical variations in parameter probability distributions. Finally, they have evaluated the relative sensitivities of (λ) to different theoretical harvest scenarios for a population of adult and sub-adult female alligators in eastern North Carolina. Their findings showed a population growth rate of 1.0156. They have observed that adult female survival had the highest sensitivity and elasticity values, while hatchling survival had the least influence on (λ). Given that adult female survival has the most significant impact on the population, we found that only a minimal total harvest of adult alligators (3%) with a low likelihood of adult females being included in that harvest (5%) would result in a scenario with a median (λ) greater than 1.

Eversole et al. (2018) stated even if there was widespread alligator (*Alligator mississippiensis*) harvesting , there has been limited scientific investigation into its impact on populations. To address this gap, a theoretical simulation model was created to evaluate the long-term effects of different harvesting strategies on alligator population trends over a 100-year period. The model, developed using system dynamics software and data from existing literature and field studies, indicated that the harvest strategy was sustainable but would result in alligator populations stabilizing below their maximum potential. The most effective strategy for maintaining a sustainable harvest while keeping alligator populations relatively stable has found to be a 38% egg harvest, 2% subadult harvest, and 2% adult harvest. Additionally, their model suggested that a higher egg harvest can be sustained if no hunting occurs, while an increased hunting harvest can be sustained without egg harvesting. Their model offers valuable insights into the role of current alligator harvesting within populations and serves as a tool for assessing the impact of changes in harvest practices or life-history characteristics on alligator population dynamics in the future.

The alligator industry in Louisiana has not been studied for thirty years to measure its economic impact. The industry has changed structurally, producing more alligator skins but with fewer producers. With potential bans on alligator sales in states like California. Their study measured the economic contribution of American alligator across the supply chain in Louisiana using four different modeling strategies. The results have shown that alligator farming in Louisiana generated a direct economic contribution of \$80,201,270 based on a Low Revenue approach for class sizes for alligator skins. When analyzing the High Revenue approach, a direct contribution of \$100,189,697 was identified. Additionally, the total economic contributions ranged from \$217 million to \$272 million to the Louisiana economy using the Analysis-by-Parts – Survey RPCs strategy. Furthermore, additional information about the distribution of spending increases the Baseline Multiplier size by 26%, and combining a more accurate distribution of spending and its location increases it by 53%. The inclusion of local spending patterns measured an additional $75.4 million in Output Effects from the alligator industry's perspective. (Rosales Chiessa, 2021)

Thomas et al. (2022) investigated the decline of the Suwannee alligator snapping turtle (*Macrochelys suwanniensis*), found exclusively in the Suwannee River drainage in Georgia and Florida. Although previous studies have examined the distribution, size, and structure of *M. suwanniensis* populations, there was limited information about its overall population status. The objectives of their study were to 1) determine population size, 2) estimate apparent survival, and 3) model population growth rates (k) by conducting a capture–mark–recapture study of *M. suwanniensis* in the Suwannee River in Florida. Over the period from 2011 to 2013, they conducted repeated sampling at 12 randomly selected 5-km sites along the Suwannee River using baited hoop-net traps, resulting in the capture of 126 individuals with 29 recaptures. Both adult males and females showed high apparent survival rates (0.99), while juveniles had lower apparent survival (0.32). They have calculated a population density of 6.6 turtles per river km, estimating a population of 1709 (95% CI, 1205–2694) *M. suwanniensis* from (approximately 259 river km). They have developed two postbreeding census matrix population models for *M. suwanniensis* using parameters from their study and literature. Both models indicated a slightly declining population trend (k = 0.99), but due to uncertainties in their estimates, they considered the population trend to be unclear. Elasticity analysis showed that k was most sensitive to changes in adult survival compared to other model components, raising concerns about incidental killing of adult *M. suwanniensis* by fishing gear.

Garcia-Grajales et al. (2012) investigated growth rates of wild American Crocodiles (*Crocodylus acutus*) of the Oaxacan coast from 2000–2009. They have assessed the age of crocodiles at the study site based on their Total length(TL), using growth rates and the von Bertalanffy model. Growth rates for TL and body weight were 0.056 ± 0.049 cm/day (n=45) and 1.092 ± 0.47 g/day(n=16), respectively. Highest growth rates in length did not always have that the highest growth rates in weight. For the von Bertalanffy model, they have used growth rate data for 23 individuals with mean TL from 700 to 1352mm between capture and recapture. Thus, the model was only applied only to young individuals. Based on model estimates, American Crocodiles from the coast of Oaxaca are larger than crocodiles of the same age from two other sites in Mexico (Banco Chinchorro and Puerto Vallarta). Although results of this study seem to agree with patterns found in other regions for this species, it is necessary to evaluate the factors influencing growth of *C. acutus* inhabiting Oaxaca's coast, especially salinity, environmental temperature variations, and precipitation.

Some of the authors above analyzed growth models for a population of crocodiles by viewing in different aspects. In this thesis we want to analyze the growth model for AMCR and control the amount of food we give to crocodiles so that we can minimize the time it takes to reach the desired length to slaughter.

CHAPTER THREE
Research Methodology

3.1 Study area

The Nile crocodile (*Crocodylus niloticus*) is widely distributed in the southern half of Ethiopia. Arba Minch (Ganta Garo) is a city and a separate woreda in the southern part of Ethiopia. Arba Minch means "40 springs" originated from the presence of more than 40 springs. Arba Minch Crocodile Ranch is located south west of Lake Abaya adjacent to Arba Minch Airport 3.8 Km far from Arba Minch town, in the Gamo zone of the Southern Ethiopia regional state, about 505 kilometers south of Addis Ababa at an elevation of 1285 kilometers above sea level. Arba Minch is the largest town in the Gamo zone.
There are two crocodile farms in Ethiopia Arba Minch Crocodile Ranch (AMCR) and Blen Development Private Limited Company (PLC) (private crocodile ranch in Arba Minch) but this study has considered secondary data from AMCR.

3.2 Mathematical Procedure

To achieve the objective of this research, we have followed the following steps.

- Reviewed related literatures.
- Formulated the mathematical model.
- Collected data from Arba Minch Crocodile Ranch.
- Solved the model (the problem) numerically using Python software.
- Developed an optimal control strategy.
- Analyzed the results of the optimal control simulations to understand how different control strategies influence crocodile growth.

CHAPTER FOUR
Mathematical Modeling of Aspect of Crocodile Ranching

4.1 Model Formulation and Analysis

The measurement of an individual's size within a specific age group is a crucial indicator of productivity in any group of animals as it affects their survival, reproduction, and population dynamics (Unterauer et al., 2021). In this thesis we have modeled the growth of a crocodile length (l) with respect to time (t) using three different growth mathematical models which are widely used in ecology the Von bertalanffy growth model (4.1), Logistic growth model (4.2) and Bertalanffy Putter(BP) growth model (4.4). And we have proposed a fourth one to overcome the short comings of these three based on Bertalanffy Putter growth model.

In this thesis, we have explained each of these models, and compare their performance in modeling the growth of a crocodile. We have used empirical data on crocodile growth. The following figures 4.1 and 4.2 shows the raw data to fit the models and evaluate the goodness of fit. We have fitted the first two from (Furstenburg, 2008) and the second from AMCR. We want to emphasize that this research depends on individual crocodile growth rather than population growth.

Table 4.1: Variables and Parameters with their description.

Variables and parameters	Description	Measured in
l	Length	Meter
t	Time	Year
k	Growth rate	-
K	Maximum length	Meter
p	The rate at which the quantity l changes	-
q	The rate at which l is being lost from crocodile	-

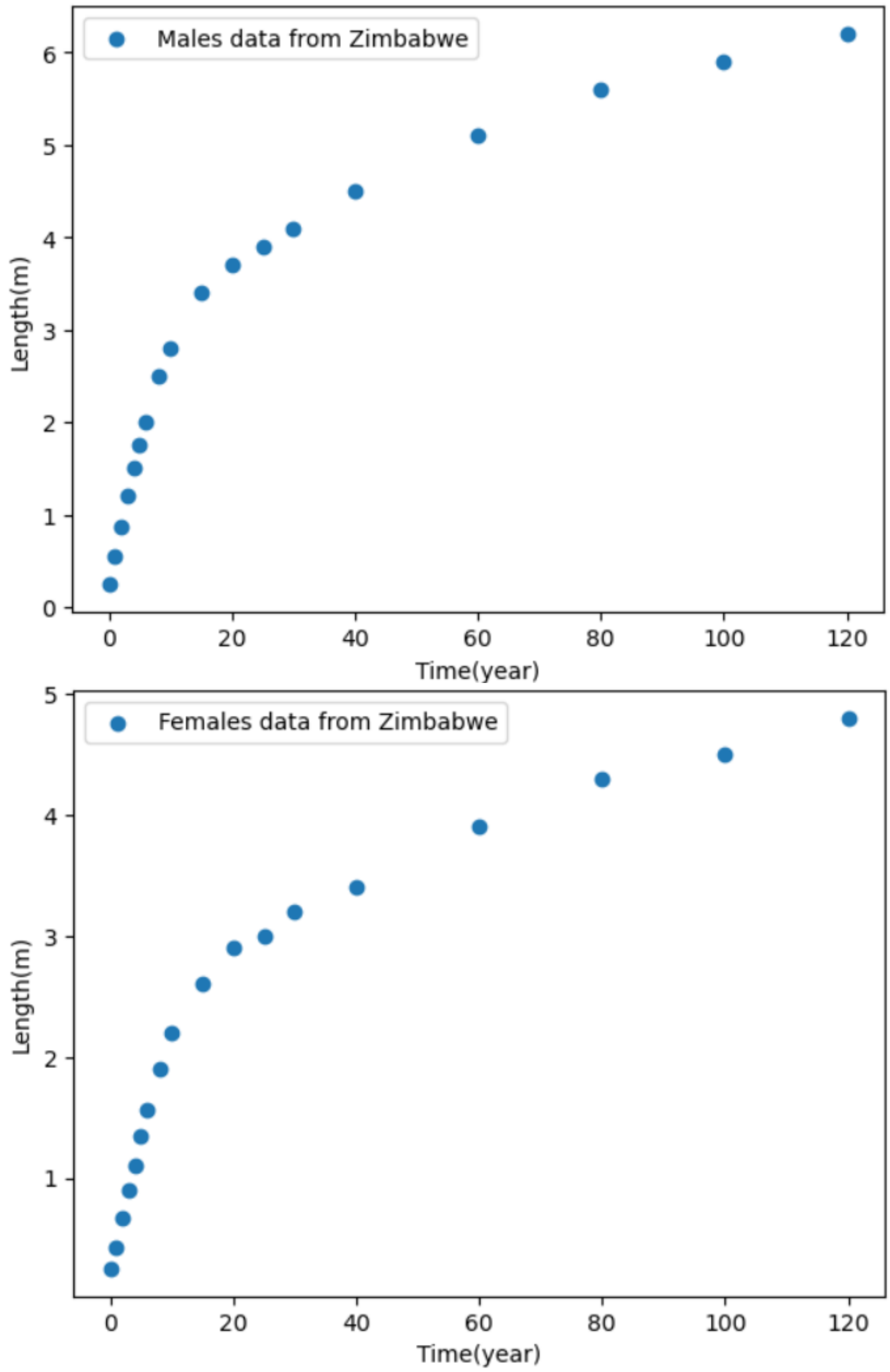

Figure 4.1: Relationship between length and age for males and females in Zimbabwe

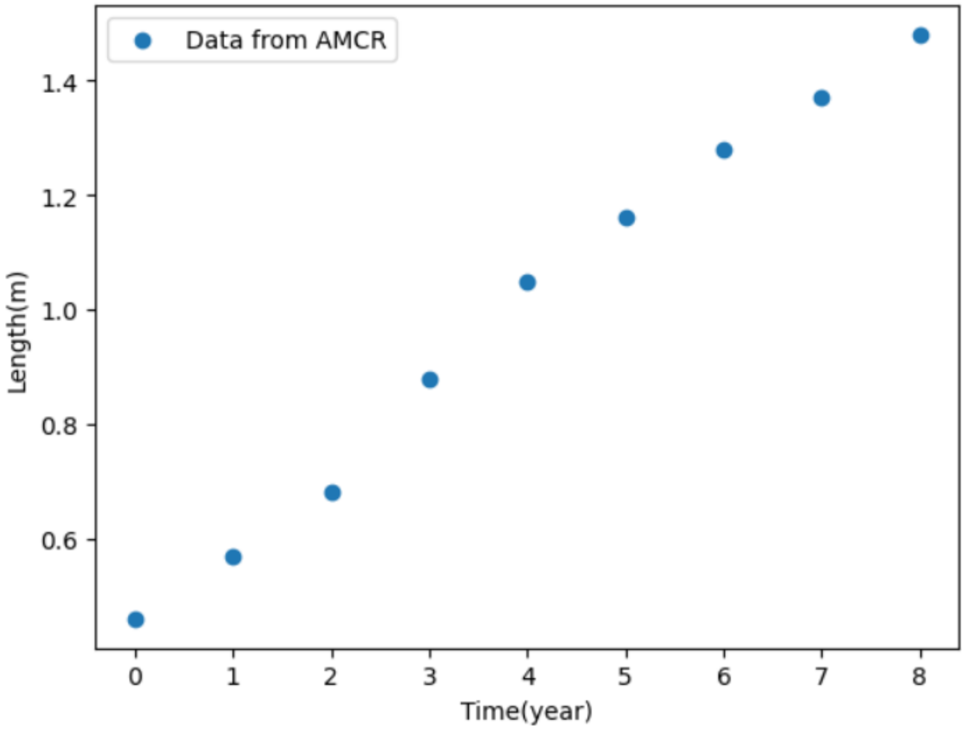

Figure 4.2: Relationship between length and age from AMCR

4.2 Von Bertalanffy growth model

Von Bertalanffy (1938) introduced an equation to study the growth of individuals belonging to several types of animal populations as

$$\frac{dl}{dt} = k(K - l), \tag{4.1}$$

with the solution $l(t) = K(1 - \exp(-k(t - t_0)))$. Where k is the growth rate (dimensionless), K carrying capacity(m), l length of crocodile(m) and t time(year).

The equilibrium point is a point where the rate of change $\frac{dl}{dt} = 0$ where the growth will be constant for equation (4.1) it can be calculated as

$$\frac{dl}{dt} = k(K - l),$$
$$\frac{dl}{dt} = 0 \quad \text{when } k = 0 \quad \text{or} \quad l = K.$$

4.3 Logistic growth model and its assumptions

4.3.1 Logistic growth model

The logistic curve was introduced by Raymond Pearl and Lowell Reed in 1920 (Pearl & Reed, 1920) and was highly known as a description of human and animal population growth (Kingsland, 1982). Which takes into account both the intrinsic growth rate k and carrying capacity of the environment (K). This study only consider the individual growth of a crocodile and the term carrying capacity as the maximum length of the crocodile.

$$\frac{dl}{dt} = kl(1 - \frac{l}{K}), \tag{4.2}$$

$l(0) = l_0$ with the solution

$$l(t) = \frac{K}{1 + (\frac{K-l_0}{l_0})e^{-kt}}. \tag{4.3}$$

4.3.2 Model Analysis

The equilibrium point is a point where the length is neither growing nor declining.

$$\frac{dl}{dt} = kl(1 - \frac{l}{K}) = f(l)$$

$$\frac{dl}{dt} = 0 \quad \text{when}$$

$$l = 0 \quad \text{or} \quad l = K$$

$$\text{from} \quad f(l) = kl(1 - \frac{l}{K})$$

$$f'(l) = k - \frac{2kl}{K}$$

$$f'(0) = k \quad \text{and} \quad f'(K) = -k$$

✓If $k > 0$, then the equilibrium point $l = 0$ is unstable and $l = K$ is stable.
✓If $k < 0$, then the equilibrium point $l = 0$ is stable and $l = K$ is unstable.
✓From the solution $l(t) = \frac{K}{1+(\frac{K-l_0}{l_0})e^{-kt}}$, we can see that as time approaches infinity the length of crocodile approaches to maximum length.

4.4 Bertalanffy Putter Growth Model Description and Its Assumptions

4.4.1 Bertalanffy growth model

The growth function $l(t)$ of the Bertalanffy-Putter (BP) model describes length, l, at time, t. It is a solution of the differential equation (4.4), which can be solved analytically, though in general not by means of elementary functions (Ohnishi et al., 2014). This equations describe body length $l(t)$ as a function of age t, using four model parameters that are to be determined from fitting the data to the model by least square method and an initial condition: The exponent pair a and b with $a < b$ and the constants p and q are non-negative, and $l_0 > 0$ is an initial value; i.e., $l(0) = l_0$. The typical solutions are increasing, bounded, and sigmoidal (S-shaped). Initially the rate of growth increases, until the inflection point is reached. Subsequently, it decreases to zero until in the limit the asymptotic mass is reached. However, for exceptional exponents and parameters, the growth curves may be non-sigmoidal or unbounded (Kühleitner et al., 2019).

4.4.2 Model Assumptions

The Bertalanffy-Pütter growth model, which describes the growth of an organism over time, is based on several assumptions. Here are some general assumptions that are often associated with this model:

- The crocodile's growth rate is proportional to its current size, raised to the power of one exponent (a).
- The crocodile's growth rate is also proportional to another factor, raised to the power of a second exponent (b). This factor could be related to the crocodile's metabolism, environment, or other factors that affect its growth.
- The crocodile's growth rate is affected by the difference between the two terms on the right-hand side of the equation: the first term represents the growth-promoting factor, while the second term represents the growth-inhibiting factor.
- The crocodile's size at time zero is a given constant ($l(0) > 0$).

4.4.3 Model Analysis

Having the Bertalanffy Putter model

$$\frac{dl}{dt} = pl(t)^a - ql(t)^b \tag{4.4}$$

The value of the parameter a is typically less than the value of the parameter b, which means that the growth rate reaches its maximum earlier than it would if the rate of deceleration of the growth rate(q) were constant. The equilibrium point for equation (4.4) can be calculated as:

$$\begin{aligned}
\frac{dl}{dt} &= pl(t)^a - ql(t)^b \\
0 &= pl(t)^a - ql(t)^b \\
\text{when } l(t) = 0, \frac{dl}{dt} &= 0 \\
pl(t)^a &= ql(t)^b \\
\frac{p}{q} &= \frac{l(t)^b}{l(t)^a} \\
\frac{p}{q} &= l(t)^{b-a} \\
l(t) &= (\frac{p}{q})^{\frac{1}{b-a}} \\
\text{where} \quad a &\neq b
\end{aligned}$$

From the above calculation we have two equilibrium points $l(t) = 0$ and $l(t) = (\frac{p}{q})^{\frac{1}{b-a}}$. Then calculating the Jacobian matrix we get

$$\begin{aligned}
F &= pl(t)^a - ql(t)^b \\
J &= \frac{\partial F}{\partial l} \\
&= pal(t)^{a-1} - qbl(t)^{b-1} \\
&= pa\frac{l(t)^a}{l(t)} - qb\frac{l(t)^b}{l(t)} \\
\text{for } a, b < 1 \quad l(t) \neq 0, \text{ thus } J_{(0)} &= \text{undefined} \\
J_{(\frac{p}{q})^{\frac{1}{b-a}}} &= pa\left(\frac{p}{q}\right)^{\frac{a-1}{b-a}} - qb\left(\frac{p}{q}\right)^{\frac{b-1}{b-a}} \\
\text{Let } * = \left(\frac{p}{q}\right)^{\frac{1}{b-a}} & \text{ be the equilibrium point}
\end{aligned}$$

$b \neq a$ for $\frac{1}{b-a}$ to be defined to calculate when $J_* > 0$

$$pa\left(\frac{p}{q}\right)^{\frac{a-1}{b-a}} > qb\left(\frac{p}{q}\right)^{\frac{b-1}{b-a}}$$
$$pa\left(\frac{p^{\frac{a-1}{b-a}}}{p^{\frac{b-1}{b-a}}}\right) > qb\left(\frac{q^{\frac{a-1}{b-a}}}{q^{\frac{b-1}{b-a}}}\right)$$
$$pa(p^{\frac{a-b}{b-a}}) > qb(q^{\frac{a-b}{b-a}})$$
$$pap^{-1} < qbq^{-1}, \text{which implies}, a < b.$$

After the above calculation we can conclude that when $a < b$, $J_* > 0$, the equilibrium point will be a stable equilibrium point (*), when $a > b$, the equilibrium point (*) will be unstable. If $a = b$, then $a - b = 0$ the equilibrium point (*) is saddle point.
Where stable equilibrium point indicates that the system will return to that equilibrium point after experiencing small perturbations or disturbances.
In the case of unstable equilibrium point small perturbations of disturbances to the system cause it to move further away from the equilibrium point and is slightly perturbed it will not return to the original equilibrium point but instead move away from it over time.

4.5 Model description

Our aim here is to show the three models described above are related to one another.

$$\frac{dl}{dt} = k(K - l) \tag{4.5}$$
$$\frac{dl}{dt} = kl(1 - \frac{l}{K}) \tag{4.6}$$
$$\frac{dl}{dt} = pl^a - ql^b \tag{4.7}$$

with a straight forward change of variables and letting $a = 0$ and $b = 1$ these can be written as

$$\frac{dl}{dt} = k(1 - \frac{l}{K}) \quad \text{by letting} \quad \frac{k}{K} \to k \tag{4.8}$$
$$\frac{dl}{dt} = kl(1 - \frac{l}{K}) \tag{4.9}$$
$$\frac{dl}{dt} = kl^a(1 - \frac{l^b}{K}) \tag{4.10}$$

Thus the Von Bertalanffy model and the logistic model are special instances of the BP model.

Here we would like to emphasize two points:
✓The Von Bertalanffy model and the logistic model are special cases of the Bertalanffy putter model.
✓The Bertalanffy Putter model ,as presented in (Kühleitner et al., 2019) is not a suitable form.This can easily be seen as the coefficients p and q have dimensions which depends on a and b, parameter that has to be defined later.The equation can be reformulated as follows:

$$\frac{dl}{dt} = pl^a - ql^b = pl^a(1 - \frac{q}{p}l^{b-a}) \tag{4.11}$$

Thus as long as $a \neq b$, $\frac{dl}{dt}$ has two equilibrium points , $l^* = 0$ and $l^* = (\frac{q}{p})^{\frac{1}{b-a}}$. Denote $\frac{p}{q}$ as K thus $l^* = (\frac{1}{K})^{\frac{1}{b-a}} = K^{\frac{1}{a-b}}$. Obviously the dimension of K is the same as of l, and $\frac{l}{K}$ is a dimensionless variable. Thus equation 4.11 can be rewritten as

$$\frac{dl}{dt} = pK^a(\frac{l}{K})^a(1 - (\frac{l}{K})^{b-a})$$

Finally, denote $pK^a = \eta$, and the equation becomes

$$\frac{dl}{dt} = \eta(\frac{l}{K})^\alpha(1 - (\frac{l}{K})^\beta) \qquad \alpha = a, \quad \beta = a - b \tag{4.12}$$

then the Bertalanffy putter model has been written in a form with correct dimensions:
l : length $[m]$
t : time[year]
K : saturation point (maximum length $[m]$)
η : growth coefficient$[\frac{m}{year}]$
α, β: dimensionless
The two simpler models are special cases of equation (4.12)

Von Bertalanffy :

$$\alpha = 0, \beta = 1, \eta = kK \Rightarrow \frac{dl}{dt} = \eta(1 - \frac{l}{K}) \tag{4.13}$$

Logistic model:

$$\alpha = 1, \beta = 1, \eta = kK \Rightarrow \frac{dl}{dt} = \eta(\frac{l}{K})(1 - \frac{l}{K}) \tag{4.14}$$

Modified Bertalanffy Putter:

$$\frac{dl}{dt} = \eta(\frac{l}{K})^\alpha(1 - (\frac{l}{K})^\beta) + c_1 l + c_0 \tag{4.15}$$

which suggests that the models are mathematically similar and can be used to describe similar growth patterns under certain conditions.

4.6 Model Analysis

Clearly, equation (4.12) has two equilibrium points, where $\frac{dl}{dt} = 0$, that is $l^* = 0$ and $l^* = K$.

Let's study this separately:

$$\left.\begin{array}{l} l^* = 0 \quad \text{Let} \quad l(t) \approx 0, \quad \text{and} \quad l(t) > 0. \quad \text{Then} \quad l'(t) > 0 \\ \qquad\quad \text{Let} \quad l(t) \approx 0 \quad \text{and} \quad l(t) < 0, \quad \text{then} \quad l'(t) < 0 \end{array}\right\} \text{only for} \quad \beta > 0$$

Thus l^* is an unstable equilibrium point, independent of α and β.
$l^* = K$; and assume $l(t) \approx l^*$ then

$$\text{If } \beta > 0 \quad \text{and} \begin{cases} l(t) > K \implies l'(t) < 0 \\ l(t) < K \implies l'(t) > 0 \end{cases} \implies \text{stable equilibrium}$$

For $\alpha > 0, \beta < 0$ we get

$$l' = \eta(\frac{l}{k})^{\alpha} - \eta(\frac{l}{K})^{\beta+\alpha}$$

$$\text{If } \beta + \alpha > 0 \quad \text{then} \begin{cases} l^* = 0 \text{ is a stable equilibrium} \\ l^* = K \quad \text{is an unstable equilibrium} \end{cases} \tag{4.16}$$

If $\beta + \alpha < 0$ or $\alpha < 0$, f is singular for $l^* = 0$ and the Peano existence can not be applied to prove existence. Because it requires continuity in l and t to prove existence of the solution.

Theorem 4.1 *Existence and uniqueness of the solution Let $t_0 > 0$ and the initial conditions satisfies $l(0) > 0$ in the prescribed region Ω_c. Then the solutions of the model system 4.17 exists and unique in $\mathbb{R}$.*

Proof.

$$\begin{aligned} \frac{dl}{dt} &= \eta(\frac{l}{K})^{\alpha}(1 - (\frac{l}{K})^{\beta}) + c_1 l + c_0, \\ f(t, l(t)) &= \eta(\frac{l}{K})^{\alpha}(1 - (\frac{l}{K})^{\beta}) + c_1 l + c_0, \\ \frac{\partial f}{\partial l} &= \eta\alpha(\frac{1}{K^{\alpha}})l^{\alpha-1}(1 - (\frac{l}{K})^{\beta}) - \eta(\frac{l}{K})^{\alpha}\beta(\frac{1}{K^{\beta}})l^{\beta-1}) + c_1. \end{aligned} \tag{4.17}$$

Here $f(t, l(t))$ has a continuous partial derivative with respect to each state variable in

$\mathbb{R}_+$ and is bounded .Thus f is locally Lipschitz in $\mathbb{R}_+$. Hence the result is the direct consequence of the Fundamental Existence and Uniqueness Theorem. Which shows the existence and uniqueness of the solutions of (4.12), (4.13) and (4.14) since we can derive all these from the model (4.17) and $\eta, \alpha, \beta, c, K$ are finite numbers thus $\frac{\partial f}{\partial l}$ is a finite number which can be expressed as $\frac{\partial f}{\partial l} \leq L$ where L is some constant which we can take as Lipschitz constant.

4.7 Modified Bertalanffy Putter Growth Model

When we think of crocodile growth their growth is linear so we need to modify the BP model to include their linear growth. Where $c_1 l$ represent the growth rate of crocodile depends on the current size of the crocodile and c_0 represent the term which affects growth rate but does not depend on current size of the crocodile. The term $c_1 l + c_0$ ensures some growth even when the size dependent terms might be negligible.

$$\frac{dl}{dt} = pl^a - ql^b + c_1 l + c_0. \tag{4.18}$$

4.8 Parameters Estimation

Least square methods produce the estimated parameters with the highest probability (maximum likelihood) of being correct (Menke, 2015). It is basically a statistical technique used to find a mathematical equation that describes the relation ship between data points as closely as possible and a line(curve) that best fits a set of data points. We have found the parameters in Table (4.2) using least square methods on python Jupyter notebook.

Table 4.2: Parameters and their estimated values for the four models

Parameters	Male Furstenburg	Female Furstenburg	Parameters of AMCR
Von Bertalanffy growth model			
η	3.00402e-1	2.25092e-1	1.61708e-1
K	5.65266	4.35586	5.02961
α	1e-10	1e-10	1e-10
β	1.0	1.0	1.0
$c0$	0.0	0.0	0.0
$c1$	0.0	0.0	0.0
Logistic growth model			
η	1.68315	1.13986	5.95373e-1
K	4.79112	3.72228	1.73466
α	1.0	1.0	1.0
β	1.0	1.0	1.0
$c0$	0.0	0.0	0.0
$c1$	0.0	0.0	0.0
Von Bertalanffy Putter growth model			
η	6.49053	5.57021e-2	2.94439e+1
K	6.04750	4.67042	1.62159
α	2.47392	4.83551e-45	1.69945
β	2.54216e-2	2.19948e-4	2.46191e-2
$c0$	0.0	0.0	0.0
$c1$	0.0	0.0	0.0
Modified Bertalanffy Putter growth model			
η	1.76241	2.33421	3.0289e+1
K	8.88971e-1	8.05194e-1	1.97408e-1
α	2.68544e-1	2.96418e-1	5.92166e-1
β	5.14468e-1	4.94089e-1	1.67438e-1
$c0$	-4.60953e-1	-8.48997e-1	-6.99422
$c1$	8.99317e-1	1.34958	3.1855875e+1

4.9 Comparison of the models

The following figures are figures that we made from the data of the crocodiles we obtained from (Furstenburg, 2008) as shown in Figure (4.1). Where the paper studied crocodiles in depth including their behavior, habitat requirement, feeding, reproduction, and the like. And he recorded age versus length data from the Zimbabwe Crocodile Farm which contained more data and long-term data than we got from AMCR Figure (4.2).

Table 4.3: Model representation with the corresponding model name

Model representation	Model name
Model 0	Von Bertalanffy model
Model 1	Logistic model
Model 2	Bertalanffy Putter model
Model 3	Modified Bertalanffy Putter model

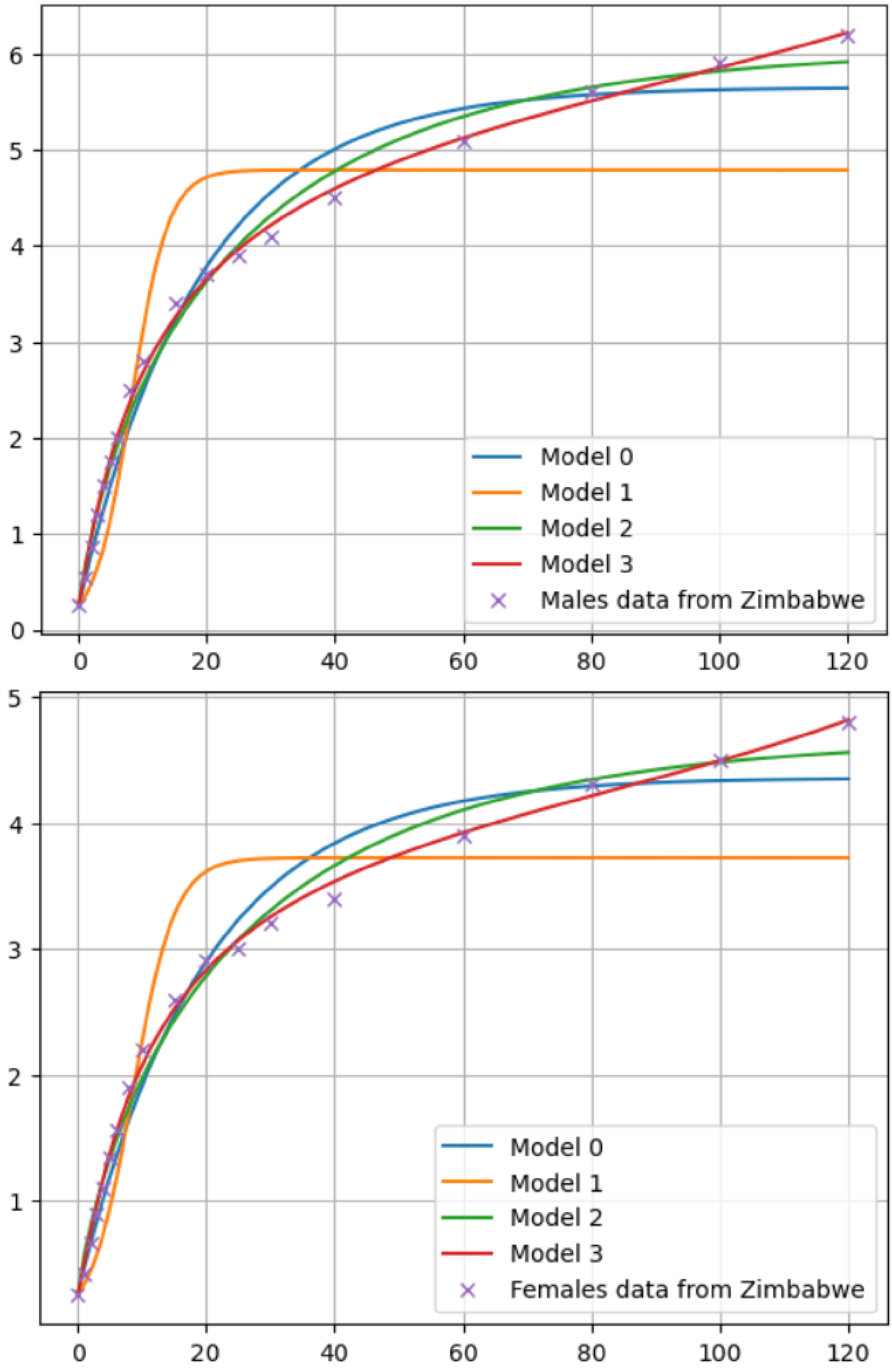

Figure 4.3: Fitted model for the data from Zimbabwe

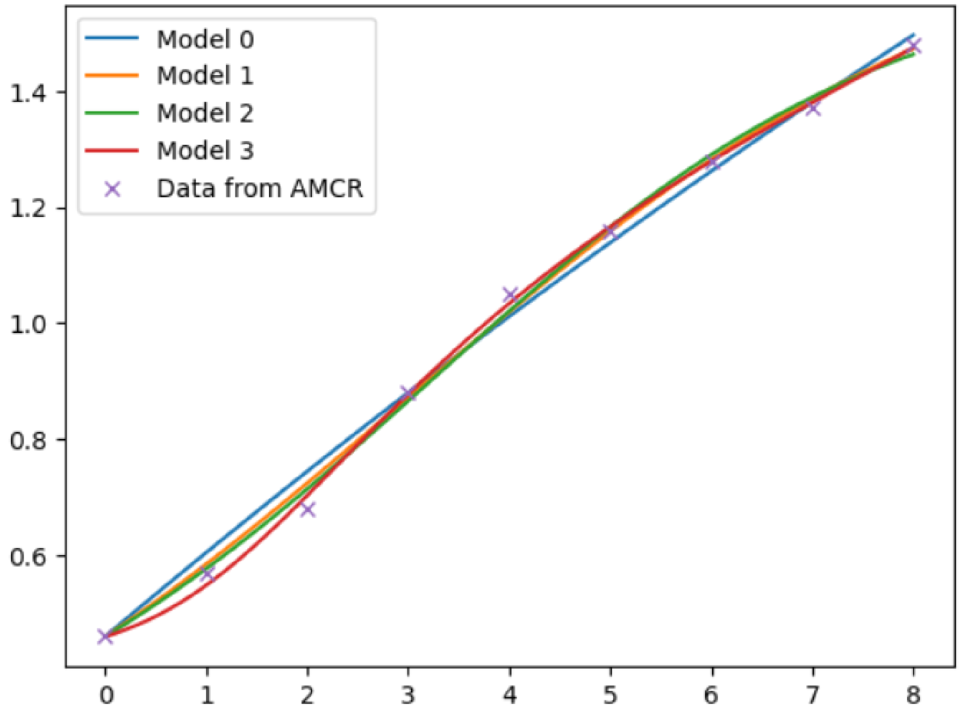

Figure 4.4: Fitted model for the data from AMCR

Table 4.4: Sum of squared differences of the models

Type of the Model	Sum of squared differences
Males (Furstenburg, 2008)	
Von Bertalanffy growth model	7.80298958802314e-01
Logistic growth model	4.456395449236858e+00
Bertalanffy Putter growth model	2.5262092215700116e-01
Modified BP growth model	**6.124797963058615e-02**
Females (Furstenburg, 2008)	
Von Bertalanffy growth model	4.435563457398686e-01
Logistic growth model	2.314541982149864e+00
Bertalanffy Putter growth model	1.7959855243373174e-01
Modified BP growth model	**4.396764275887459e-02**
AMCR	
Von Bertalanffy growth model	4.016764386488271e-03
Logistic growth model	1.7791606052935782e-03
Bertalanffy-Putter growth model	1.529428035079337e-03
Modified BP growth model	**7.204134841744612e-04**

4.9.1 Comparison of the models with respect to their sum of squared differences AMCR

As we can see from Table (4.4) above Modified Bertalanffy Putter model is the best fit for all long term data and short term data.

CHAPTER FIVE

Mathematical Modeling of Aspect of Crocodile Ranching With Optimal Control Analysis

5.1 Optimal Control Analysis

5.2 Time Optimal Control

Optimal control is concerned with finding the best possible way to control a system given a specific objective and constraints. Optimal control has applications in many fields like engineering, economics, biology and other fields, where precise control over system behavior is important (Boscain & Piccoli, 2005).
Time optimal control focuses on finding the control strategy that moves a system from a starting state to a desired final state in the shortest possible time. It's basically about achieving a specific outcome with a system as quickly as possible. The key elements are :
Dynamics function in optimal control problem is defined as a dynamics function g that describes the evolution of a system over time (Macki & Strauss, 2012).
Cost functional $J(l, u)$, is defined over the entire trajectory l and a control input u. It often involves integrating a running cost function $L(l(t), u(t), t)$ over time where L is the instantaneous cost and J is the total cost.
Control inputs are the adjustable elements we can manipulate to influence the system's behaviour.
The objective is given the dynamics function and cost functional, the goal is to find a control strategy that optimizes the cost functional $J(T, l(.), u(.))$ while satisfying dynamics function and constraints.
By optimizing these elements, optimal control strategies can increase system performance, improve effectiveness, and enable the fulfillment of specific goals (Udrişte, 2008).
There are two ways of choosing a control:
Open loop when we choose u as a function of time t (Macki & Strauss, 2012).
Closed loop when we choose u as function of space variable l (Macki & Strauss, 2012).
Mathematically optimal control can be expressed as given an optimal control problem with:

Let f, g, ϕ be $\in C^1$ with respect to all of their variables t, l and u, and consider the following free terminal problem (T is not fixed) :

$$\text{Cost functional} \quad J(T, l(.), u(.)) = \phi(T) + \int_0^T f(t, l(t), u(t))dt \tag{5.1}$$

$$\text{State equation} \quad l'(t) = g(t, l(t), u(t)), \quad t \in [0, T], \tag{5.2}$$

$$l(t_0) = l_0, \quad l(T) = l_d, \quad u(t) \in PC \in \Omega \tag{5.3}$$

Several mathematical techniques like the Pontryagin Maximum Principle, dynamic programming, and optimization algorithms play an important role in solving optimal control problems (Evans, 2005).

Theorem 5.1 *Pontryagin Maximum principle for free terminal time problem If the triple (T^*, l^*, u^*) is solution to the free terminal time optimal control problem (5.1), then there exist multipliers λ_0 and $\lambda(.)$, not all zero where λ_0 is 0 or 1 for maximization problems (5.1) and 0 or -1 for minimization problems (5.1) and $\lambda \in PC^1([t_0, T]; \mathbb{R}^n)$, such that the following conditions hold: (Torres, 2021)*

the minimality condition

$$H(t, l^*(t), u^*(t), \lambda(t)) = min_{u \in \Omega} H(t, l^*, u, \lambda(t)); \tag{5.4}$$

the Hamiltonian system

$$\begin{cases} \frac{dl^*(t)}{dt} = \frac{\partial H}{\partial \lambda}(t, l^*(t), u^*(t), \lambda(t)) \\ \frac{d\lambda(t)}{dt} = -\frac{\partial H}{\partial l}(t, l^*(t), u^*(t), \lambda(t)) \end{cases} \tag{5.5}$$

where the Hamiltonian is defined by

$$H(t, l, u, \lambda) = \lambda_0 f(t, l, u) + \lambda g(t, l, u). \tag{5.6}$$

with the boundary condition

$$\lambda(T) = \nabla \phi(T) \tag{5.7}$$

Moreover,

$$H(T^*, l^*(T^*), \lambda(T^*)) = -\lambda_0 \phi'(T^*) \tag{5.8}$$

Then for all $t \in [0, T]$ we have

$$u^* = argmin_{u \in \Omega} H(l^*(t), u, \lambda(t)) \tag{5.9}$$

5.3 Feeding of Crocodiles

The rate of growth is how efficiently the crocodile is utilizing its environment and will be directly affected by food availability, food quality, food intakes, season and sex of animal (Games, 1990). The dietary habits of *Crocodylus niloticus* undergo a shift over its life span, similar to the ontogeneic shift observed in other crocodile species. The Ontogenetic shift in crocodiles refers to the changes in their diet and feeding behavior as they grow and develop from juveniles to adults (Wallace, 2006). Juveniles consume insects, spiders, snails and mussels in shallow waters and on the shore. As they grow, their diet progressively include primarily amphibians like toads and frogs,and small fish. Sub-adult crocodiles start to feed more on reptiles, including terrapins, pythons, snakes, water-birds, crabs, and rodents. As they reach adulthood, their diet mainly consists of barbel (catfish) and large mammals, including young giraffes, buffalo, and elephants. They have been observed attacking and consuming plains zebras, Cape buffalo, warthogs, hyenas, baboons, blue wildebeests, impalas, giraffes, big cats (particularly lions), and even other crocodiles (Furstenburg, 2008). But this feeding style is not for the crocodiles in the captive rearing as they do not prey .
In AMCR crocodiles food is mainly composed of byproduct of fish and red meat where the amount and composition depends on their age Table 5.1 ,5.2. There are around 3290 crocodiles in AMCR Figure 5.1 shows age composition of the farm.

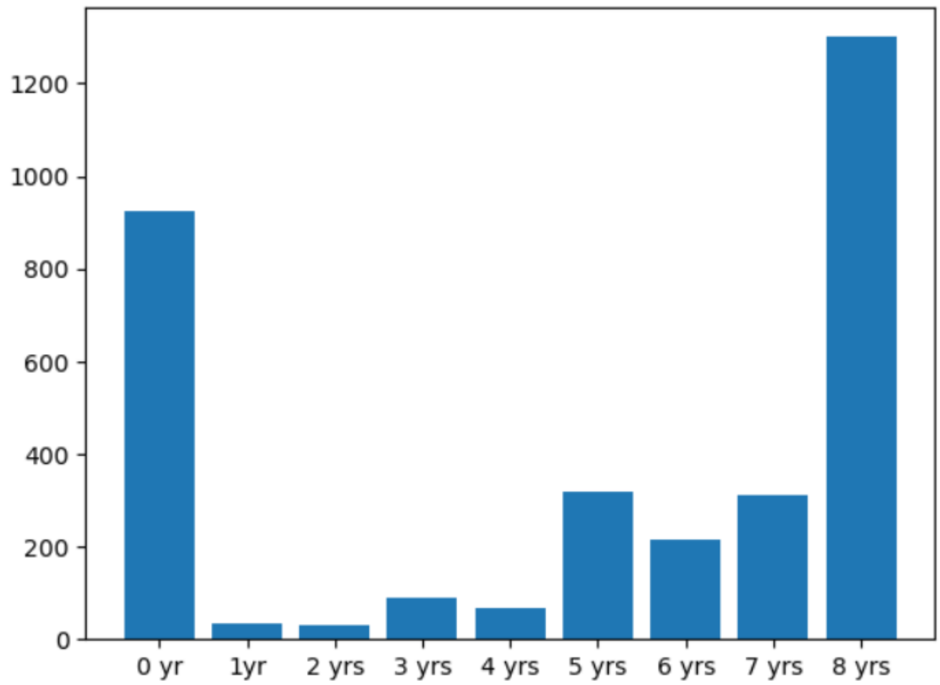

Figure 5.1: Age composition of the farm in AMCR

5.3.1 Feeding of the crocodiles in Arba Minch Crocodile Ranch

Table 5.1: Required food composition of crocodiles from 0-3 years

No	Type of byproduct	required composition amount
1	fish\fish byproduct	60%
2	red meat \meat byproduct	30%
3	ground blood and bone	9.5%
4	vitamins and minerals	0.5%

Table 5.2: Required food composition of crocodiles from 4-8 years

No	Type of byproduct	required composition amount
1	fish\fish byproduct	65%
2	red meat \meat byproduct	35%

In this chapter we will propose an optimal control framework on Logistic growth model, where the objective function is to minimize the time it takes for one crocodile to reach the target length $l_d(T) = 1.2m$ (Furstenburg, 2008) given the total amount of food over the lifespan.

On the month when this data was collected from AMCR, the temperature averaged a high of 32°C and a low of 19°C. This leads to a better digestive system for the crocodiles, causing them to want to eat more, which is supported by (Wallace, 2006). Since the crocodiles in the ranch cannot go out and prey for themselves, their growth is very limited, besides many other factors.
AMCR was governed by the federal government of Ethiopia from its establishment in 1976 E.C. to 2008 E.C. Since then, it has been governed by the regional government of Southern Nations, Nationalities and Peoples Region (SNNPR) and administered under the Regional Bureau of Culture and Tourism. This has led to many financial and export problems, which are affecting their growth.
Last year, crocodile skin was not exported, but this year they are negotiating to export it, including the meat for the first time. However, until the export agreement is made, AMCR plans to slaughter the crocodiles and send the skin to Ethiopian Leather so that they can preserve it there, because feeding all those crocodiles is difficult.

Lake Abaya which is approximately 6km far from AMCR, demolished the first AMCR in 2008, leading to lose of many crocodiles and an economic crisis (lost buildings, office, cars and pools including electric pools for the hatchlings). Very good control is given for the crocodiles less than 3 years including feeding vitamin and minerals since their survival rate is 17% (Tosun, 2013).

Table 5.3: Age versus length data from Zimbabwe Versus AMCR

Age	Male Crocodile Zim.	Female crocodile Zim.	Crocodile AMCR
Birth	0.25	0.25	0.46
1 year	0.55	0.42	0.57
2 years	0.87	0.67	0.68
3 years	1.2	0.9	0.88
4 years	1.5	1.1	1.05
5 years	1.75	1.35	1.16
6 years	2.0	1.56	1.28
8 years	2.5	1.9	1.48

Table 5.4: Comparison between amount of food in AMCR and mean amount of food for Nile Crocodile stated in Frustenberg(2008) in Kg.

Age	Amount to feed in a year(AMCR)	Mean amount to feed	Deviation
0	6.24	7.8	1.56
1	9.36	18.2	8.84
2	14.36	26	1.64
3	20.8	20.8	0

Crocodiles until the age of 2 years can change 50% of the food to their body. For example if you feed one crocodile 2kg, 1kg will be added to their weight. Relatively, for cattle, sheep and pigs to increase the same mass they need to eat 3-5 folds. As a study in Nile crocodile shows for a one crocodile to finish the food which weighs as its total mass it will take 125-160 days.

5.4 Extension of the model into optimal control

The goal is to minimize the objective functional J by increasing the amount of food since it is reported that 50% of the food crocodiles eat is changed in to their body when less than 2 years old. We can see length differences between AMCR and a crocodile farm in Zimbabwe of the same species (*Crocodylus niloticus)* (Furstenburg, 2008) in Table 5.3. The age of slaughtering crocodiles is 3 years in Australia (*Crocodylus porosus*) (Isberg et al., 2005) and Zimbabwe (*Crocodylus niloticus*) (Furstenburg, 2008). 1.5m in Zambia in 2.5 years (*Crocodylus niloticus*) (Dzoma et al., 2008). But in AMCR the minimum age is 6 years with length 1.2m. And we have noticed the amount they feed in AMCR is less than the mean weekly feeding of a Nile crocodile stated in (Furstenburg, 2008) which affects growth significantly beside other factors. We can see the difference in Table 5.4. In this part of the thesis we have tried to minimize the time to reach a certain length by increasing the amount of food we feed once or increase the frequency of the amount of food in the life span.

Mathematically, the optimal control problem consists of minimizing the objective functional equation 5.10, the time used to reach a certain length for the same amount of food in life span (8 years) 170.36 kg.
We define the cost functional for the minimization problem as

$$\begin{aligned} J[u(t)] &= \int_0^T (1 + u^2(t))dt \to \min \\ \text{where} \quad T & \quad \text{is free terminal time} \\ \text{and} & \quad u \text{ is the amount of food} \end{aligned} \tag{5.10}$$

Subjected to

$$\frac{dl}{dt} = \eta(\frac{ul}{K})(1 - \frac{l}{uK}) \quad l(0) = 0.46 \quad l_d(T) = 1.2 \tag{5.11}$$

$$J(u^*) = min\{J(u)\} : u \in \Omega \tag{5.12}$$

where $u = 1$ current state
$u > 1$ more food
$u < 1$ less food

5.5 Characterization of the Optimal Control Solution

We have desired crocodile length $l_d(T) = 1.2$ meters
Current food intake through life span (8 years) 170.36 kilograms.
Food intake function: $F(t) = 7.22 + 2.93t$ which represents the yearly food intake found by least square method.
Length growth function: $\frac{dl}{dt} = \eta(\frac{l}{K})(1 - \frac{l}{K})$ which represents the rate of change of length ($\frac{dl}{dt}$) based on the current length(l), growth rate constant (η), maximum attainable length(m) (K).
Objective: Minimize the time (T) it takes for the crocodile to reach a desired length ($l_d(T) = 1.2$) meters, using the same or less food than it is currently receiving (i.e total food intake over T years) $\leq$ current intake for 8 years).
Control variable $u(t)$ represents the yearly food intake the crocodile receives. Which can be adjusted within a range (possible limitation on food availability)
Control input constraint the yearly food intake cannot exceed a certain limit u_{max}.

$$u_{min} \leq u(t) \leq u_{max}$$

Cost function

$$J[u(t)] = \int_0^T (1 + u^2(t))dt \to \min \tag{5.13}$$

Modified system dynamics: we introduce the control variable $u(t)$ to directly affect the rate of change of the crocodile length $(\frac{dl}{dt})$. Which reflects the idea that the control variable influences how much the food actually contributes to growth.
The modified system dynamics becomes

$$\frac{dl}{dt} = \eta(\frac{ul}{K})(1 - \frac{l}{uK}) \tag{5.14}$$

Here taking AMCR as our concrete example and taking the values we got from fitting the data $\eta = 0.59, K = 1.73$ and solving for $l(t)$ we get

$$l(t) = \frac{l_0 uK}{l_0 + (uK + l_0)e^{\frac{-\eta ut}{K}}}$$

Let T be the age where the crocodiles are slaughtered. Now if we substitute desired length $l_d(T)$ in place of $l(t)$ we can easily solve for T.

$$l_d(T) = \frac{l_0 uK}{l_0 + (uK + l_0)e^{\frac{-\eta uT}{K}}}$$

where $l_d(T)$ is the desired length

solving for T we get

$$T = \frac{-K}{\eta u} \ln \left| \frac{l_0 uK - l_d l_0}{l_d uK - l_d l_0} \right|$$

Since the desired length $l_d(T)$, l_0 length at age 0, the maximum length K and η are known the only unknowns here will be T and u.
If we do a simple analysis on the formula T from the numerator we have

$$l_0 uK - l_d l_0 > 0$$

$$uK > l_d \implies u > l_d/K$$

substituting the values of l_d and K we get

$$u > 0.69$$

Again from the denominator

$$l_d uK - l_d l_0 > 0$$

$$u > l_0/K$$

$$u > 0.26$$

So we have $u > 0.26$ and $u > 0.69$ which we can conclude as $u \geq 0.7$. Thus we have found the minimum $u_{min} = 0.7$ and we took u_{max} to be 2 to represent the amount of

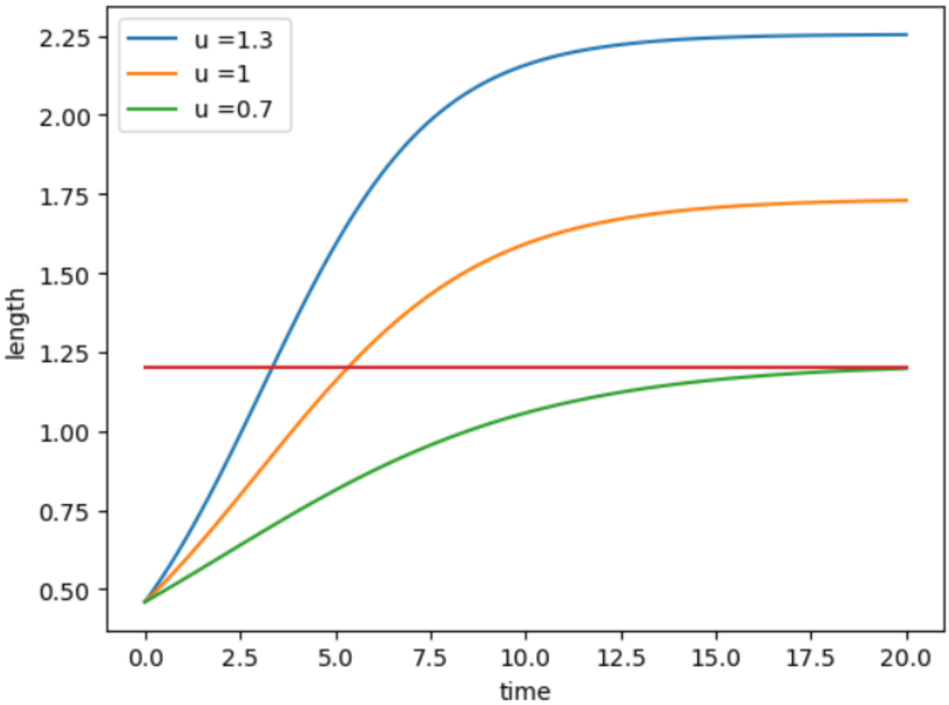

Figure 5.2: The effect of increasing amount of food on length

food to be doubled.
The following figure shows how varying the value of u will change the time to reach the desired length $l_d(T) = 1.2m$. The red line represents when we can reach the desired length with different value of u.
The objective function will attain its minimum when $u = 1.58$.
In Figure 5.3 we can see that how the change of u value (the amount of food) can affect the slaughtering age T.

As we can see from the above figure as we increase the amount of food u the slaughtering age T will decrease. We have found the minimum time to be $T = 1.81$ when u is doubled.
If we just take the objective function $J = \int_0^T udt$ we would get Bang system hitting the minimum at $u = 2$ 5.5

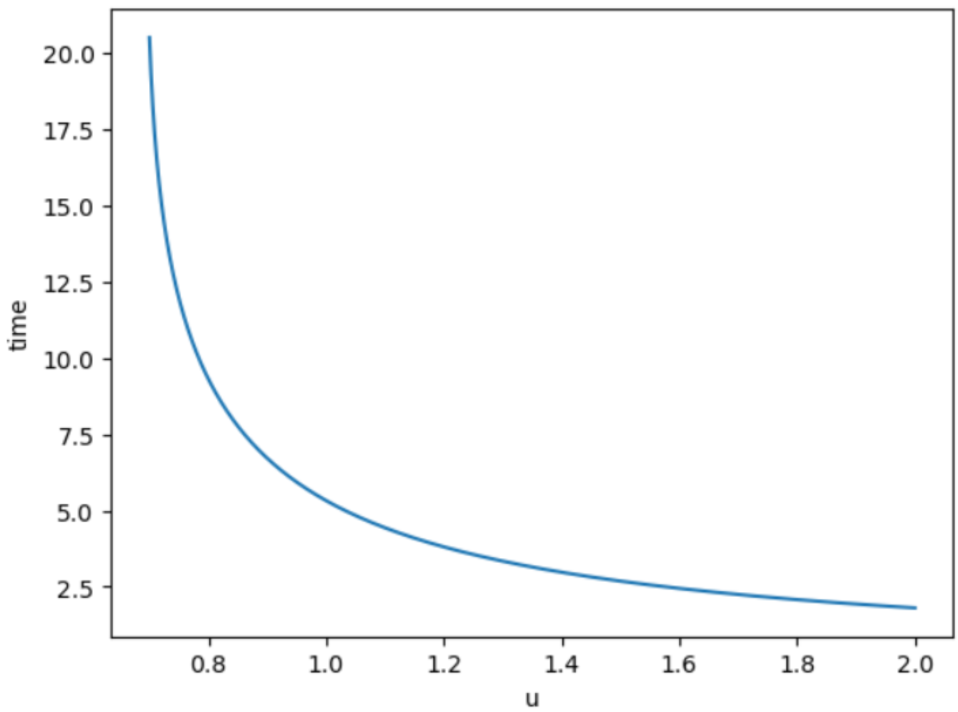

Figure 5.3: The effect of increasing amount of food on slaughtering age

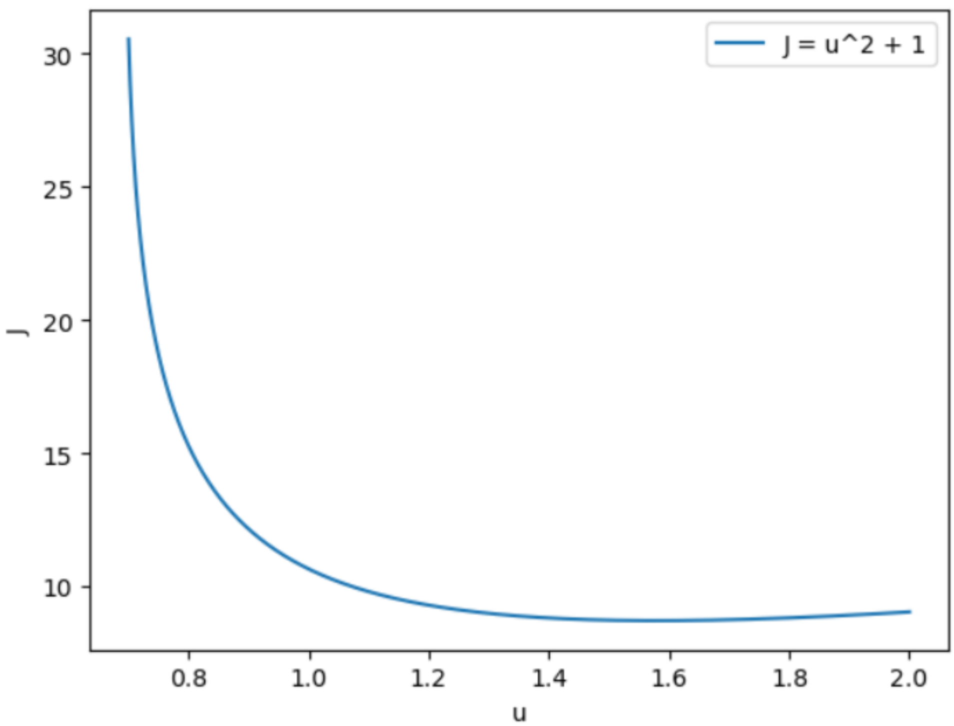

Figure 5.4: The minimum value for the objective function

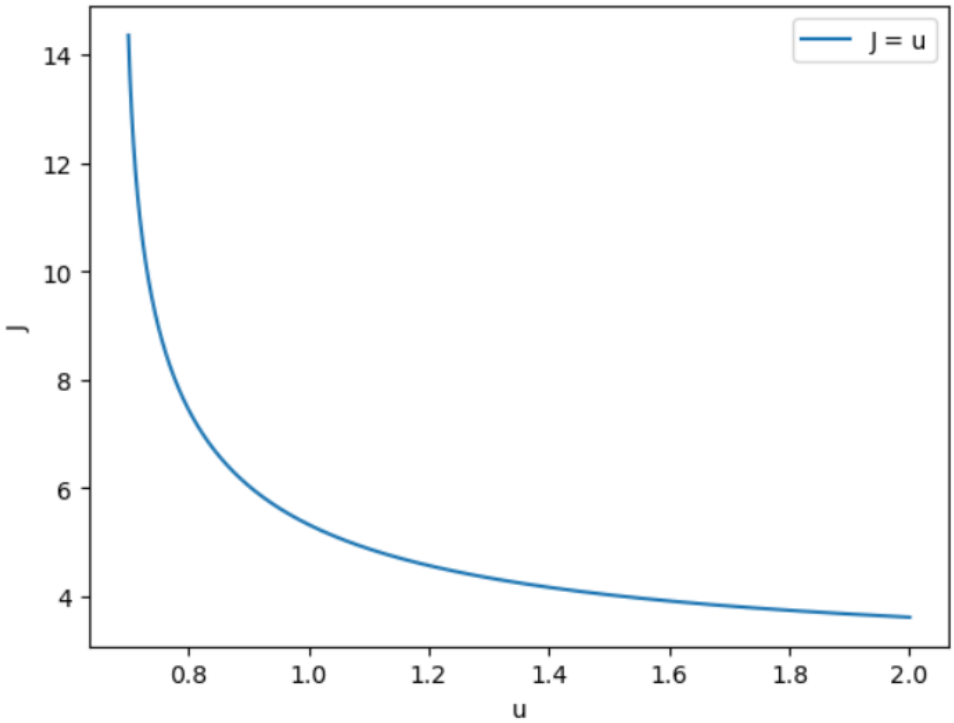

Figure 5.5: The minimum value for the objective function (u = 2)

Conclusion and Recommendation

Conclusion

Crocodile farming is an essential industry since its skin is a valuable asset for textile and luxury industry and plays an important role in conserving crocodile population. In this thesis, we have developed a mathematical model based on von Bertalanffy Putter growth model to study the dynamics of crocodile ranch as a case study in Arba Minch Crocodile Ranch and compared it with three different growth models namely von Bertalanffy growth model, Logistic growth model, von Bertalanffy Putter growth model all three with sigmoidal curves. And have found that the modified Bertalanffy model is the best fit to model the growth of a crocodile with a small value of sum of squared differences for both data from AMCR and (Furstenburg, 2008). The model can be adopted and applied to other crocodile species and farms, making it a valuable resource for the conservation and management of these species. Our model takes into account the current size, growth promoting factor and a growth inhibiting factor. This study serves as a model for future research in this area, highlighting the importance of overall approach to understanding and managing crocodile populations. After AMCR has started operating in 1985 one other private crocodile ranch was also developed, Blen development private limited company. Both farms are in the southern parts of Ethiopia. The original objectives for the farm were conservation of Nile crocodile in Lakes Abaya and Chamo, commercializing crocodile skin and meat to the global market and tourism in the country. Now a days most of the income is from tourism which implies there should be done an overall work in skin production and selling of meat. AMCR was started mainly by the capture of crocodiles, current stock includes 926 hatchlings and 2364 growers. From all the simulations made in chapter 5 we can conclude that by increasing the amount of food we can decrease the years we spend by feeding the crocodiles since we can make them reach the desired length earlier.

Recommendation

Having the capacity to ranch crocodiles in Ethiopia with a species *Crocodylus niloticus* and moderate temperature in most of the places of the country it can be done better. The data can be collected by demography as such things would help researchers. Expansion of nearby lake Abaya threatens to flood the ranch, as happened in 2008 insuring the ranch is now in safe site should be a priority.

To decrease the amount of time, money, and energy spend in feeding the crocodiles up to minimum six years we can increase the amount of food given in a year so that we can reach to the desired length as earlier as possible. Increasing the amount of food should not be that difficult as (Tyowua & Uloko, 2019) recommended frog feeds should be main diet of juvenile which we can get easily from building small ponds near the ranch. Or building other farms like sheep, goat or hen parallel to AMCR would help crocodiles as the byproduct would be the food for crocodiles.

Possible future work

This study can be extended to investigate the impact of climate change, nutritional constraint, hydration constraint and the likes on the growth of Nile crocodiles in Arba Minch Crocodile Ranch. Additionally the model can be used to explore the potential impact of different management strategies on the crocodile population.

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Appendix

Python code for models (4.12, 4.13, 4.14, 4.15)

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import least_squares
from scipy.integrate import solve_ivp


# Models
def BP_modified(t, l, eta, K, alpha=0.0, beta=1.0, c0=0.0, c1=0.0):
    # This is the most general model.
    # vB: alpha=0, beta=1, c0=0, c1=0
    # logistic: alpha=1, beta=1, c0=0, c1=0
    # BP: c0=0, c1=0
    if alpha<0:
        alpha = 0.0
    if beta<0:
        beta = 0.0
    if K<0.0:
        print('K', K)
    if l<0.0:
        print(t, 'l ', l)
        l = 0

    return eta*(l/K)**alpha*(1-(l/K)**beta)+c1*l+c0


# The f-function for the least squares method
def model_func(x, t, y, model=0):
    '''
    Construct the function for the least square method.
    model=0 : von Bertalanffy
    model=1 : logistic
    model=2 : von Bertalanffy Putter
    model=3 : von Bertalanffy Putter + linear growth
    '''
```

```
    eta = x[0]
    K = x[1]
    alpha = x[2]
    beta = x[3]
    c0 = x[4]
    c1 = x[5]
    l0 = y[0]

    if model == 0:
        f = lambda t, l: BP_modified(t, l, eta, K, alpha=0.0)
    elif model == 1:
        f = lambda t, l: BP_modified(t, l, eta, K, alpha=1.0)
    elif model == 2:
        f = lambda t, l: BP_modified(t, l, eta, K, alpha=alpha, beta=beta)
    elif model == 3:
        f = lambda t, l: BP_modified(t, l, eta, K, alpha=alpha, beta=beta,
    sol= solve_ivp(f,[0, t[-1]], [l0], t_eval=t, atol=1.e-6, rtol=1.e-4)
    assert sol.status != -1, "ODE solver fails"
    return sol.y[0,:]-y


def present_solution(model, res, t, l0):
    '''
    Plot the solution of a model for the parameters given by x at the poin
    '''
    eta = res.x[0]
    K = res.x[1]
    alpha = res.x[2]
    beta = res.x[3]
    c0 = res.x[4]
    c1 = res.x[5]
    f = lambda t, l: BP_modified(t, l, eta, K, alpha=alpha, beta=beta, c0=
    sol= solve_ivp(f,[0, t[-1]], [l0], t_eval=t, atol=1.e-6, rtol=1.e-4)
    plt.plot(sol.t, sol.y.T,  label=f'Model {model}')

    print(f'Results for model {model}')
    print(f'Success : {res.success}')
```

```
print(f'Parameters:')
print(f'    eta   = {res.x[0]}')
print(f'    K     = {res.x[1]}')
print(f'    alpha = {res.x[2]}')
print(f'    beta  = {res.x[3]}')
print(f'    c0    = {res.x[4]}')
print(f'    c1    = {res.x[5]}')

print(f'Least square error (squared): {res.cost}')
```

References

Abate, G. (2022). Threats of nile crocodile in lake chamo and the risk factors for arba minch crocodile ranch, gamo zone, arba minch, ethiopia. *Fisheries Livest Prod 10: 368.*, *40*, 368.

Adan, R. (2000). Crocodile farming: a multi-million dollar industry. *SEAFDEC Asian Aquaculture*, *22*(3), 22–23.

Arega, T., Ayano, A. A., & Tintaku, T. (2022). Assessment of major health problem and constraints of crocodile skin production at arba minch crocodile ranch.

Aust, P. W. (2009). *The ecology, conservation and management of nile crocodiles crocodylus niloticus in a human dominated landscape* (Unpublished doctoral dissertation). Division of Biology, Imperial College London.

Bolton, M. (1989). *The management of crocodiles in captivity* (Tech. Rep.). FAO.

Boscain, U., & Piccoli, B. (2005). An introduction to optimal control. *Contrôle non linéaire et applications, Herman, Paris*, 19–66.

Chabreck, R. H., & Joanen, T. (1979). Growth rates of american alligators in louisiana. *Herpetologica*, 51–57.

Chala, G., Aychiluhim, T. B., & Karthikeyan, M. (2020). A systematic study on nile crocodile (crocodylusniloticus) skins for the preparation of leather products. *International Journal of Development Research*, *10*(08), 39605–39614.

Delene, K., Lemma, A., & Fesseha, H. (2020). Major diseases of nile crocodile (crocodylus niloticus) with focus on current status in arba minch crocodile ranch, ethiopia. *Online Journal of Animal and Feed Research*, *10*(3), 98–110.

Dembner, S. (1990). Fao assistance in crocodile management efforts. *Unasylva*, *41*(161), 21–25.

Dzoma, B., Sejoe, S., & Segwagwe, B. (2008). Commercial crocodile farming in botswana. *Tropical animal health and production*, *40*, 377–381.

Evans, L. C. (2005). *An introduction to mathematical optimal control theory.* University of California.

Eversole, C. B., Henke, S. E., Turner, B. L., Glasscock, S. N., Powell, R. L., Wester, D. B., & Ballard, B. M. (2018). A theoretical population and harvest model for american alligators (alligator mississippiensis). *Herpetological monographs*, *32*(1), 22–33.

Fergusson, R. A. (2010). Nile crocodile crocodylus niloticus. *Crocodiles. Status Survey and Conservation Action Plan, 3rd edn (eds Manolis SC, Stevenson C)*, 84–89.

Furstenburg, D. (2008). Nile crocodile crocodylus niloticus (laurenti, 1768).

Games, I. (1990). Growth curves for the nile crocodile as estimated by skeletochronology. In *Proceedings of the 10th working meeting of the crocodile specialist group. species survival commission of the international union for the conservation of nature and natural resources, gainesville, florida* (pp. 111–121).

Garcia-Grajales, J., Buenrostro-Silva, A., Charruau, P., et al. (2012). Growth and age of juvenile american crocodiles (crocodylus acutus) in la ventanilla estuary, oaxaca, mexico. *Herpetological Conservation and Biology*, *7*(3), 330–338.

Garner, L. A., et al. (2016). Population status of the american alligator (alligator mississippiensis) in north carolina.

Isberg, S., Thomson, P., Nicholas, F., Barker, S., & Moran, C. (2005). Quantitative analysis of production traits in saltwater crocodiles (crocodylus porosus): Ii. age at slaughter. *Journal of Animal Breeding and Genetics*, *122*(6), 370–377.

Jacobsen, T., & Kushlan, J. A. (1989). Growth dynamics in the american alligator (alligator mississippiensis). *Journal of Zoology*, *219*(2), 309–328.

Kamien, M. I., & Schwartz, N. L. (1991). *Dynamic optimization, volume 31 of advanced textbooks in economics.* North-Holland Publishing Co., Amsterdam,.

Kingsland, S. (1982). The refractory model: The logistic curve and the history of population ecology. *The Quarterly Review of Biology*, *57*(1), 29–52.

Kühleitner, M., Brunner, N., Nowak, W.-G., Renner-Martin, K., & Scheicher, K. (2019). Best-fitting growth curves of the von bertalanffy-pütter type. *Poultry Science*, *98*(9), 3587-3592. Retrieved from `https://www.sciencedirect.com/science/article/pii/S0032579119306510` doi: https://doi.org/10.3382/ps/pez122

Lenhart, S., & Workman, J. T. (2007). *Optimal control applied to biological models.* CRC press.

Lindsey, P. A., Barnes, J., Nyirenda, V., Pumfrett, B., Tambling, C. J., Taylor, W. A., & Rolfes, M. t. (2013). The zambian wildlife ranching industry: scale, associated benefits, and limitations affecting its development. *PloS one*, *8*(12), e81761.

Macki, J., & Strauss, A. (2012). *Introduction to optimal control theory.* Springer Science & Business Media.

Maria, A. (1997). Introduction to modeling and simulation. In *Proceedings of the 29th conference on winter simulation* (pp. 7–13).

Masser, M. P. (1993). Alligator production: an introduction. *Pub.(Louisiana Cooperative Extension Service)(USA)*(2538).

Mendal, J. (2012). South african farms aim for bigger, better crocodile skin. *Pan American Leathers*, *22*.

Menke, W. (2015). Review of the generalized least squares method. *Surveys in Geophysics*, *36*, 1–25.

Ohnishi, S., Yamakawa, T., & Akamine, T. (2014). On the analytical solution for the pütter–bertalanffy growth equation. *Journal of Theoretical Biology*, *343*, 174–177.

Pearl, R., & Reed, L. J. (1920). On the rate of growth of the population of the united states since 1790 and its mathematical representation. *Proceedings of the national academy of sciences*, *6*(6), 275–288.

Revol, B. (1995). Crocodile farming and conservation, the example of zimbabwe.

Rootes, W. L., Chabreck, R. H., Wright, V. L., Brown, B. W., & Hess, T. J. (1991). Growth rates of american alligators in estuarine and palustrine wetlands in louisiana. *Estuaries*, *14*, 489–494.

Rosales Chiessa, I. F. (2021). Implementing four modeling improvement strategies to measure the economic contribution of the alligator farming industry in louisiana.

Saalfeld, K., Fukuda, Y., Duldig, T., & Fisher, A. (2016). Management program for the saltwater crocodile (crocodylus porosus) in the northern territory of australia, 2016-2020. *Northern Territory Department of Environment and Natural Resources, Darwin.*

Sangha, K. K., Stoeckl, N., Crossman, N., & Costanza, R. (2019). A state-wide economic assessment of coastal and marine ecosystem services to inform sustainable development policies in the northern territory, australia. *Marine Policy*, *107*, 103595.

Shirely, M., Siege, L., & Ademasu, M. (2014). Crocodile management in ethiopia. *Crocodile Specialist Group.*

Stickney, R. R. (2000). *Encyclopedia of aquaculture.*

Thomas, T. M., Enge, K. M., Suarez, E., & Johnston, G. R. (2022). Population status of the suwannee alligator snapping turtle (macrochelys suwanniensis) in the suwannee river, florida. *Chelonian Conservation and Biology: Celebrating 25 Years as the World's Turtle and Tortoise Journal*, *21*(1), 2–10.

Tisdell, C. A., & Swarna Nantha, H. (2005). Management, conservation and farming of saltwater crocodiles: an australian case study of sustainable commercial use.

Torres, D. F. (2021). A brief introduction to optimal control theory with python. *Mathematics of Control, Signals and Systems*.

Tosun, D. D. (2013). Crocodile farmi ng and its present state in global aquaculture. *Journal of FisheriesSciences. com*, *7*(1), 43–57.

Tucker, A. D., Limpus, C. J., McDonald, K. R., & McCallum, H. I. (2007). Growth dynamics of freshwater crocodiles (crocodylus johnstoni) in the lynd river, queensland. *Australian Journal of Zoology*, *54*(6), 409–415.

Tyowua, B., & Uloko, I. (2019). Growth performance of juvenile nile crocodile (crocodylus niloticus) fed on chicken fillet, fish and frog.

Udrişte, C. (2008). Multitime controllability, observability and bang-bang principle. *Journal of Optimization Theory and Applications*, *139*, 141–157.

Unterauer, H., Brunner, N., Kühleitner, M., et al. (2021). Modelling the growth of rearing cattle. *Czech Journal of Animal Science*, *66*(11), 441–449.

Viva.org.uk. (2024). How crocodiles are farmed and killed. *https://viva.org.uk/animals/other-animals/crocodiles/*.

Von Bertalanffy, L. (1938). A quantitative theory of organic growth (inquiries on growth laws. ii). *Human biology*, *10*(2), 181–213.

Wallace, K. M. (2006). *The feeding ecology of yearling, juvenile and sub-adult nile crocodiles, crocodylus niloticus, in the okavango delta, botswana* (Unpublished doctoral dissertation). Stellenbosch: University of Stellenbosch.

Whitaker, R., & Whitaker, N. (2008). Who's got the biggest. *Crocodile Specialist Group Newsletter*, *27*(4), 26–30.

Wilkinson, P. M., & Rhodes, W. E. (1997). Growth rates of american alligators in coastal south carolina. *The Journal of Wildlife Management*, 397–402.