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**FACULTY OF ELECTRICAL AND ELECTRONICS TECHNOLOGY AND
INFORMATION AND COMMUNICATION TECHNOLOGY
(DEPARTMENT OF ELECTRICAL AND ELECTRONICS
TECHNOLOGY)**

**Gantry Crane Position and Anti-Sway Control Using Optimal Motion
Position Regulator and Fuzzy-Tuned PID Controller**

MSc Thesis for the Partial Fulfillment of
Master of Science in Electrical Automation and Control Technology Management

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Addis Ababa, Ethiopia



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A Thesis submitted to
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FACULTY OF ELECTRICAL AND ELECTRONICS TECHNOLOGY
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In partial fulfillment for the Degree
**MASTER OF SCIENCE in ELECTRICAL AUTOMATION AND CONTROL
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By,
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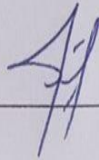
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DECLARATION

I hereby declare that the work which is being presented in this thesis entitled “Gantry Crane Position and Anti-Sway Control Using Optimal Motion Position Regulator and Fuzzy-Tuned PID Controller” is the original work of my own, has not been presented for a master’s thesis in this or other universities and all sources of materials used for this thesis work have been fully acknowledged.

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ABSTRACT

The most frequent form of crane utilized in industrial situations for transferring loads is a gantry crane. When we try to convey the weight rapidly by speeding the trolley or rail, the load sways more. This unfavorable swing must be reduced before any loading or unloading can take place, and this procedure takes time. Furthermore, the unfavorable swaying would frequently produce excessive stress on the hoist, as well as harm to the surrounding items and the payload. The gantry cranes control dilemma is how to get the cargo to its destination as rapidly as feasible without creating excessive load swinging. For the gantry crane control problem, a combination of optimum motion planer and fuzzy-tuned PID controllers are presented. Before developing a fuzzy-tuned PID controller to follow the reference signal generated by the optimum motion position regulator, the first step will be to design an optimal motion position that provides a reference command. The optimal motion position is an open-loop algorithm that generates an ideal trajectory that the gantry cranes trolley/carte should follow for optimal performance and fuzzy-tuned PID controller is used to control the sway angle and position of gantry crane. Moreover, the nonlinear nature of the fuzzy-tuned PID controller is advantageous for controlling the highly nonlinear model of the gantry crane. When adding external disturbances to the gantry crane model, the fuzzy-tuned PID control system performs better than the classical PID controller. The sway angle and angular velocity settling time (3sec and 0.3sec respectively for fuzzy-tuned PID) and also the sway angle and angular velocity settling time for PID (3.5 sec and 0.4sec respectively). Therefore, the fuzzy-tuned PID controller shows that better optimal sway angle tracking performance and better performance on the position control as compared to conventional PID controllers.

Keywords:-Gantry Crane, Fuzzy Controller, Sway Angle, Optimal Motion Position Regulator, Fuzzy-Tuned PID Controller

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LIST OF ABBREVIATIONS

2D	Two Dimensional
3D	Three Dimensional
CE	Change of Error
COG	Center Of Gravity
DOF	Degree Of Freedom
E	Error
FIS	Fuzzy Inference System
FLC	Fuzzy Logic Controller
LQR	Linear Quadratic Regulator
MPC	Model Predictive Control
PID	Proportional plus Integral plus Derivative
PSO	Particle Swarm optimization

LIST OF SYMBOLS

θ	theta (sway angle)
$\dot{\theta}$	Angular displacement
$\ddot{\theta}$	Angular acceleration
$e(t)$	error signal
$f(t)$	applied force
K_P	Proportional gain
K_I	Integral gain
K_D	Derivative gain
m	mass of payload
$r(t)$	reference signal
t_a, t_d	acceleration, deceleration time
$u(t)$	control signal
V_{f1}	maximum velocity of trolley
V	potential energy
X	distance of the cart
\dot{x}	velocity of the cart
\ddot{x}	acceleration of the cart
x_p	distance of the payload
$y(t)$	output signal

CHAPTER ONE

INTRODUCTION

1.1. Background

The first modern cranes for loading and unloading cargo were constructed at ports during the Industrial Revolution. The cranes were required to be extremely powerful due to the work they were performing. As a result, they were incorporated into stone towers in order to increase their sturdiness. Cranes, for example, went from being made of wood to being made of cast iron and steel during the industrial revolution. Cranes systems are used all over the world. Cranes are usually employed at factories, shipyards, construction sites, warehouses to transport heavy loads [1].

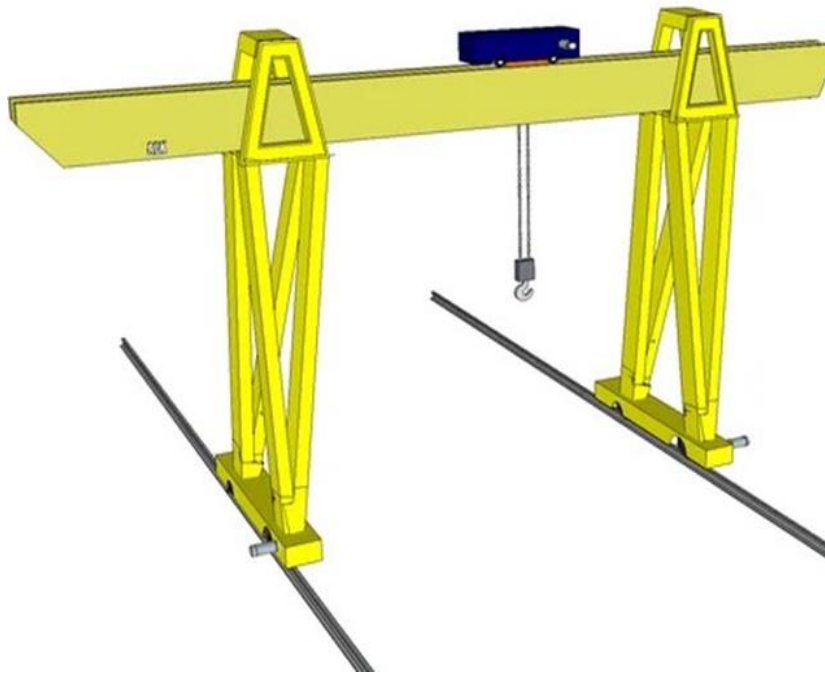


Figure 1. 1: Gantry Crane Structure [1]

The first mechanical crane was created in the eighteenth or nineteenth century, as a result of the development of steam engines. Cranes are electromechanical machines that use a hoist mechanism, a hook, and wire cables to raise and transfer large things from one location to another. Cranes are a vital part of our lives in the twenty-first century, and numerous types of cranes are employed, particularly in the manufacturing and industrial sectors.

The most prevalent cranes include tower cranes, gantry cranes, boom cranes, and bridge (overhead) cranes. The main purpose of controlling a gantry crane is to transport the load without causing any extra excessive swing angle at its destination. However, many of the gantry cranes can result of suddenly stopped of their payload when the swing motion is fast.

It will take time to schedule and then reduce the swing motion to a minimum. Additionally, to stop the swing instantly at the proper position, the gantry crane operation requires a skilled operator to manually control the sway based on his or her experiences. Different kinds and degrees of accident inhuman beings and the surrounding will occur if the control mechanism of a gantry crane is poor [2].

The classical controllers like Proportional plus derivative (PD) and proportional plus derivative (PD)-type FLCs are the techniques used to investigation to actively control the sway angle of gantry crane system but they are not good enough to control the non-linear gantry crane. A nonlinear dynamic model of the nonlinear gantry crane is considered and is derived using the Eulers-Lagrange formulation [3].

Gantry cranes are widely used to load and unload cargo, move materials in the construction industry, and assemble heavy machinery in the manufacturing sector. However, the performance of this equipment may be limited by the fact that the loads move in a pendulum-like manner, which is hazardous to industrial security.

Although an operator can reduce this detrimental motion by moving the trolley in small increments, this will result in a poor level of efficiency. Because of the small swing angle, high safety, low transit time, and high positioning precision for the gantry crane automation of this equipment's operation are desirable. Many studies on overhead crane control challenges have been published in recent decades. The systems have been made public. There are many different types of control methods. However, when it comes to crane models, there are two types of control methods to consider. A single class is used to create a linearized crane system model. The nonlinear model requires the other [4].

Gantry cranes are under-actuated systems by nature, in which several actuators are less than some degrees of freedom (DOF). The gantry crane system has a single actuator for actuating two different variables. The position of the trolley/bridge and the sway angle of the payload

are variables that are controlled by a linear force exerted by the motor on the trolley/cart or the rail. Some advantages of under-actuated systems are low cost, low power consumption, and simple structure. However, for under actuated systems, particularly for the gantry crane system becomes harder due to controlling for a single actuator with two different variables. The gantry crane is one of the most widely used cranes in the world, and it may be customized for a variety of applications. In addition, the crane's size may be scaled to practically any application, and the crane's hoist can perform a variety of tasks.

The largest gantry cranes now in use are those for transferring enormous loads, such as loading and/or unloading different size and weights of containers from boats on decks and in harbors. There are also some large gantry cranes capable of moving tens of thousands of tons that are employed in the construction of different areas of yachts, cruise ships, and oil rigs.

In fact, the world's largest cranes are gantry cranes, which have a simple yet effective design. Nowadays, the number of industry factories that utilize a gantry crane is quickly expanding since many people believe that utilizing a gantry crane for transferring big cargo from one location to another is safer, faster, and easier.

Operators of gantry cranes must raise a load and maneuver the crane from its beginning position to the desired location in practice. The operators must then wait for the swing/sway angle to vanish before loading the weight into the desired location, which takes time.

1.2. Objective

1.2.1. General objective

The general objectives of the thesis are to design an optimal motion-position regulator of the gantry crane and designing fuzzy-tuned PID Controller for position and anti-sway control of the gantry crane.

1.2.2. Specific Objectives

The specific objectives of this thesis are the following:-

- To develop a mathematical model of dynamics gantry crane system
- To design optimal motion-position regulator which will used to produce reference trajectories for the motion of the rail and the trolley/gantry crane

- To design a Fuzzy algorithm that can intelligently tune the PID gains.
- To simulate the proposed controller using the models developed in MATLAB Simulink platforms
- To compare the performance of the designed controller to a classical PID controller
- To evaluate the robustness of the proposed control system by introducing external disturbances to the system

1.3. Statement of Problem

Gantry crane is type of crane used to move the payload from one desired position to another position. During the load the gantry crane behaves like a pendulum during the transportation that process sway around the destination position. If the sway reaches a final critical limit, the crane process must be stopped until the sway disappears otherwise the situation will lead to accident, injury, or fatality. Crane operation in general requires quick transferring of the load to the desired position without excessive swaying.

Unfortunately, moving big weights quickly would cause them to sway excessively. The swaying of the load can also be influenced by system parameter uncertainty and external environmental disturbances like wind. Because of uncontrolled payload swing dynamics, it is necessary to have an anti-sway controller installed in a gantry crane system to suppress the swaying without human interaction. To optimize between fast transporting and less swaying, optimal motion tracking algorithms are a great tool to use.

Using optimal motion tracking, a desired velocity profile trajectory that should be tracked by cart/trolley will be produced. Different types of controllers can be used to better track the reference velocity profile signal by the cart or the trolley. The highly nonlinear crane dynamics are bettered controlled with a nonlinear control scheme.

That is why I propose a Fuzzy-tuned PID controller for the system since it has nonlinear nature, is highly robust to parameter variation, and exploits the benefit of both Fuzzy and PID controllers. Therefore, this research aims to develop a control system that suppresses the sway angle and improves the position accuracy of the gantry crane.

1.4. Scope and limitation of the thesis

This thesis covers the modeling of gantry crane dynamics, designing an optimal motion planner for the controlled system, designing a Fuzzy-tuned PID controller for the controlled system by assuming the output signal of the optimal motion planner as the reference. After the design of the control system, the performance will be evaluated and simulate in MATLAB and their results are compared to a controller that substitutes the Fuzzy-tuned PID part of the control system with the PID controller. There will not be any hardware implementation of the designed control system.

1.5. Significance of the Study

Since cranes are under actuated oscillatory systems their control is not a trivial task. Many types of research have been done in the area of controlling gantry cranes, aiming at the reduction of positioning errors and traveling time. Several control strategies have been put out in last decade for the management of gantry cranes, and they can essentially be divided into open-loop and closed-loop strategies. Open-loop techniques are by far the most popular due to their low cost as they do not require feedback sensors to measure the states of the crane.

Optimal motion-position regulator is the most well-known open-loop strategy due to its effectiveness but as an open-loop strategy, it best fits with when working along with other closed-loop controllers. Since the two parameters of system, the position, and swaying angle have to be controlled together, a combination of two controllers is the most used scheme for the control problem.

The combination of an optimal motion-position regulator and a closed-loop controller is a great choice of scheme for the control of gantry cranes. This is because unlike using two closed-loop controllers, the two controllers used in this scheme will not interfere with each other's work. A combination of optimal motion-position regulator and Fuzzy-tuned PID controller is proposed here.

Optimal motion-position regulator is an open-loop algorithm that will produce an ideal trajectory that should be tracked by the trolley/cart of the gantry crane for best performance. Fuzzy-tuned PID then other hands fuzzy-tuned PID is a feedback controller that considers changes in the system. This makes optimal motion-position regulator and fuzzy-tuned PID controller complements each other in solving the problem.

Moreover, the highly nonlinear model of gantry crane is better controlled with a nonlinear controller like fuzzy-tuned PID.

1.6. Methodology

To achieve the above-mentioned objectives, this thesis will contain the following major sequence of tasks several journal articles, conference papers, books, websites, etc. will be reviewed on the optimal motion-position regulator, fuzzy-tuned PID controller, and gantry crane dynamics modeling.

Mathematical modeling of gantry crane system using Euler-Lagrange method designing optimal motion-position regulator first then designing fuzzy-tuned PID controller by taking the output signal of the optimal motion position regulator as the reference input. The performance of the designed controller will be assessed.

The gantry crane has two movable parts, the trolley, and the rail. One Part can move through X-axis the other in the Y direction. Using these movements, the crane can reach any point in the X-Y Cartesian plane within the working area boundary of the crane. When a force is exerted on the trolley or the rail it accelerates causing the load attached to the hoist to swing in the air. To transport the load to its destination with less swing and fast time we will first design an optimal motion position regulator.

The optimal motion-position regulator has an algorithm to optimize time under constraints of less swing of the load and position accuracy. The input to the optimal motion-position regulator is a displacement that should be traveled by the trolley or the rail to reach their respective desired destinations. The output of the optimal motion-position regulator can be either velocity or acceleration profile signals and these signals are ideal references that should be tracked by the trolley or the cart to reach their desired destination within quick time and less swing.

Finally using the output of the optimal motion-position regulator as our reference signal we will design and a fuzzy-tuned PID controller for the gantry crane's system dynamics.

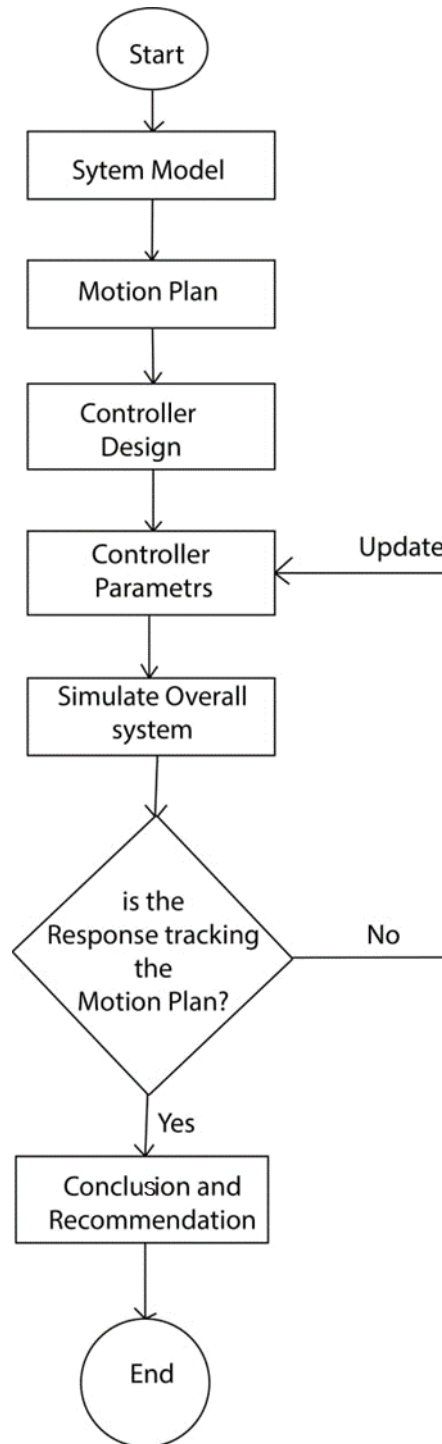


Figure 1. 2: Flow Chart of Gantry Crane Control

1.7. Thesis organization

There are six chapters in this thesis:-The first chapter includes an introduction, problem statement, research objectives, methodology, scope and limitation of the study, and significance of the study. The second Chapter presents literature review on the gantry crane control system. The third chapter discusses the mathematical modeling of two-dimensional gantry cranes and the optimal motion-position regulator based on desired position coordinate. Controller design is covered in Chapter 4.It covers fuzzy logic, fuzzy-tuned PID, and PID controller designs with the desired input reference being the ideal inputs coordinate. Chapter 5 discusses two-dimensional gantry crane systems and presents simulation results. This chapter also covers the examination and comparison of the simulation outcomes for each of the aforementioned controllers. The entire thesis is concluded in chapter six. It also contains a suggestion for additional research.

CHAPTER TWO

LITERATURE REVIEW

In the modern day, a nonlinear control system is becoming a popular research area. The gantry cranes position and sway angle control is a best example being selected as a study of a nonlinear system. This system has great application in almost all industries many scholars have been studying in this area. Among them some of articles are reviewed as follows.

Almutairi and Zribi, 2016 [5] have studied on controlling a gantry crane one has to ensure that the cart is properly positioned while minimizing the swings of the payload. Hence, the cart of the crane should move toward its destination as fast and as precise as possible while the swings of the payload should be kept as small as possible. However, the motion of the cart of the crane is always accompanied with swings of the payload. These swings can be dangerous and may cause damage or accidents. Moreover, the parameters of the crane and the payload are generally not known exactly; also external disturbances might act on the crane system. Therefore, it is necessary to develop controllers to properly control gantry cranes even when some of their parameters are not known exactly and/or when some disturbances are acting on crane. And also, the researcher does not consider the motion position regulator of the gantry crane. One can notice it is not good enough and requires further demonstration.

Omidi, 2014 [6] was proposed modeling and nonlinear control of gantry crane using feedback linearization method is presented. This thesis, a dynamic gantry crane model is developed using a powerful method called a Lagrange method. The State feedback linearization technique is applied for the model linearization and its weaknesses are reviewed. To solve these problems, the state feedback gain matrix is calculated using the Linear Quadratic Regulator method and results are fully investigated. Nonetheless, in the linearized model he used, it is difficult to investigate the nonlinear characteristics of the gantry crane. And also, the researcher does not consider the motion planning of the gantry crane. From a system response of the gantry crane, the settling time is long and the rise time is slow. One can notice it is not good enough and requires further demonstration.

Alhassan et al., 2016 [7] was proposed a gantry crane system's closed-loop position and sway control techniques are studied. In this thesis, the performance of the gantry crane control methods proportional plus integral plus derivative and linear quadratic regulator (LQR) is

examined for both payload sway control and trolley position tracking. The dynamic equation used for the gantry crane system considered is two dimensional (2D). The Lagrange energy equation is used to create the system's nonlinear model, and Taylor's series expansion is then used to linearize the system. In an approximate model in which many system parameters are assumed helps the researcher from considering the nonlinearities and removes system modeling complexity but the response is not as acceptable as the true model. Not only this, but also the motion planning or trajectory planning is not considered. Again the system response is not adequate, which means the rise time is slow and higher overshoot/undershoot is detected but the steady state error is very good.

Cakan and Onen, 2016 [8] have studied on position regulation and sway control of a nonlinear gantry crane system had proposed. With this paper, the controller used is Proportional plus derivative controller (PD) for both the gantry crane's position and sway angle, having said that, the controller gains K_P and K_D are manually tuned. Also, the motion planning is not studied. The overshoot response of the system is high, and the rising time is also slower.

Feng, Yang and Shao, 2020 [9] has worked on undesirable sway effects of the spreader of the under actuated cranes bring great challenges for control engineers. With traditional control methods, the spreader may spend much time to come to a standstill, which will significantly reduce the efficiency of port terminal. Linear quadratic regulator (LQR), which can carry out optimization by taking both position and angle into consideration, provides an adequate way for anti-sway control of the cranes. However, the performance of the LQR controller is generally poor in the presence of external disturbances and model uncertainties. The drawbacks of this thesis are Fuzzy-Tuned PID Controller is unknown further designed to compensate the undesirable effects of disturbances and uncertainties. A composite disturbance minimize controller by combining optimal motion position regulator and fuzzy-tuned PID controller is then developed for anti-sway control of the crane system subject to various uncertainties.

Zhang, Wang and Lu, 2014 [10] has worked on Gantry cranes are widely used in modern industrial applications, covering from conventional transporting heavy loads, transferring parts, and materials for manufacture lines to conveying gantry robots or functional

components in factories. Accordingly, various attempts and investigations have been carried out in gantry crane control. As unwanted swing of crane payloads causes both safety hazards and difficulty in positioning, anti-swing becomes a well-known term and research focus in gantry crane control. The drawback of this thesis is difficult to provide a mathematical description and stability proof for the FLC. Furthermore, fuzzy logic controllers (FLCs) have been applied in anti-swing control of gantry cranes for its ability to mimic the behavior of human operator accurately. As the nature of a fuzzy control system requires expert knowledge to tune the parameters, which is often difficult and time-consuming.

Abduljabbar, Billingsley and Wen, 2021 [11] have studied on investigation into the performance of Lyapunov pole placement (LPP), linear quadratic regulator (LQR) and proportional-integral-derivative (PID) control schemes for payload sway control and trolley position tracking of a gantry crane system is presented. A 2D gantry crane system is considered. The nonlinear model of the system is derived using the Lagrangian energy equation and then linearized using Taylor's series expansion.

Ospina-Henao and López-Suspes, 2017 [12] have studied on Fuzzy Controller is used for the anti-sway tracking control of overhead cranes. Fuzzy Logic Controllers have been designed to deal with problems and situations where conventional control theories have failed. Fuzzy Logic Controllers have the capability of transforming linguistic information and expert knowledge into control signals. One of the main advantages is that its implementation process is comparatively simpler than conventional methods as it works on certain set of predefined rules which are simple if-then statements which are simple to understand as they are in common English language. The input parameters after being read from the sensors are fuzzified as per the membership function of the respective variables. These membership function curves are utilized to come to a solution and finally defuzzification is done to obtain a crisp output.

Solihin et al., 2008 [13] has proposed on optimal PID controller of automatic gantry crane control using particle swarm optimization (PSO) is presented. According to Lagrange equation the dynamic model of 2-D gantry crane prototype is derived in the paper. The particle swarm optimization (PSO) method is used to automatically modify the PID gains in the proportional plus integral plus derivative (PID) controller. The good features of this thesis

are the PID controllers KP, KD, and KI are tuned automatically by the PSO algorithm. The drawbacks of this thesis are the motion trajectory is unknown which means not studied and the system response is poor. Also, the system response is not robust.

Su et al., 2009 [14] has proposed on model predictive control of gantry crane with input nonlinearity compensation studied. In the paper application of a nonlinear model predictive control (MPC) method for gantry crane system control is discussed. To adjust for the input nonlinearity, a pre-compensator is constructed using the system inverse function (non-symmetric dead zone with saturation). By fine-tuning the weighting function matrices, the control between the crane position and the swing angle is compromised. The drawbacks of this thesis are that the system's mathematical model is linear and the motion plan trajectory is not considered. The response of the gantry crane to its sway angle is not settling for a long time to its equilibrium points and the steady state is also not lowest.

Bashir, Bature and Abdullahi, 2018 [15] have studied on pole placement control of a 2D gantry crane system with varying pole locations is considered. In this paper, the system model is linearized, and the controller is a pole placement controller with three poles located at various points on the left side of the s-plane. Due to highly nonlinearity of gantry crane, controlling the position and sway using manual method of varying location of poles is not advisable criteria. Also, the model does not specify the real world's specification due to many assumptions while linearizing the model. Again, the motion planning trajectory is not considered. The system responses using pole placement method showed that a higher overshoots and longer time to settle. Therefore, nonlinear controllers are the best option for nonlinear systems.

Alhassan et al., 2015 [16] has proposed on Optimal Analysis and Control of 2D Nonlinear Gantry Crane System . In this paper presents a dynamic behavior of a nonlinear and linear model of a gantry crane system based on the system parameters. The nonlinear model was derived using Lagrange equation followed by linearization using Taylor's series approximation. MATLAB simulation results confirmed that the trolley displacement and payload oscillation are dependent on the system parameters; cable length, payload mass and trolley mass. The drawbacks of this thesis are that the system's mathematical model is linear and the motion plan trajectory is not considered.

Comparison of Active Sway Control of a Gantry Crane Systems had been Proposed by Ahmad et al., 2009 [17]. This paper presents the use of anti-sway angle control approaches for a two-dimensional gantry crane with disturbances effect in the dynamic system. Delayed feedback signal (DFS) and proportional-derivative (PD) controller are the techniques used in this investigation to actively control the sway angle of the rope of gantry crane system. A nonlinear overhead gantry crane system is considered and the dynamic model of the system is derived using the Euler-Lagrange formulation. A complete analysis of simulation results for each technique is presented in time domain and frequency domain respectively. Performances of both controllers are examined in terms of sway angle suppression and disturbances cancellation.

In similar manner “Hybrid Input Shaping and PD-type Fuzzy Logic Control Scheme of a Gantry Crane System” had been proposed by Ahmad, Nasir, et al., 2009 [18]. This paper presents investigations into the development of hybrid control schemes for input tracking and anti-swaying control of a gantry crane system. A nonlinear overhead gantry crane system is considered and the dynamic model of the system is derived using the Euler-Lagrange formulation. To study the effectiveness of the controllers, initially a collocated PD-type Fuzzy Logic control is developed for cart position control of gantry crane. This is then extended to incorporate input shaper control schemes for anti-swaying control of the system. The positive input shapers with the derivative effects are designed based on the properties of the system. Simulation results of the response of the manipulator with the controllers are presented in time and frequency domains. The performances of the hybrid control schemes are examined in terms of level of input tracking capability, swing angle reduction and time response specifications in comparison to the PD-type Fuzzy Logic control.

In research journal by Golovin, Maksakov and Palis, 2020 [19] was proposed “Gantry Crane Position Control via Parallel Feed-forward Compensator’. This article is concerned with an output feedback position control of gantry (overhead) cranes applying a parallel feed-forward compensator (PFC) that allows for a reduction of payload swinging without additional sensors or payload swing angle estimations. Performance and stability of the controlled system are achieved by defining a new output as a combination of the original output and the output of an appropriate PFC. The later provides an almost strict positive real (ASPR) condition for the

augmented plant. Thus, high gain output feedback control becomes applicable. The proposed feedback control approach is successfully validated in a numerical study and in experiments on a laboratory plant.

Aktas et al., 2018 [20] has proposed on Anti-Sway Control of a Gantry Crane with LMI Based Robust Pole Placement: Experimental Verification for Acceleration Control Approach. In this study, anti-sway control of an experimental overhead crane system is proposed with acceleration control approach. In this approach, acceleration of the cart is considered as control input. In order to satisfy transient performance objectives in the presence of parameter variations, a robust pole placement controller is designed. The parameter variations are modeled with a linear poly topic model. Hence, controller design is formulated as a convex optimization problem under linear matrix inequalities (LMIs) constraints.

2.1. Summary of Literature Review

From the literature reviewed above, the papers are used approximated mathematical model of gantry crane but gantry cranes are highly nonlinear system and no consideration of motion trajectory of the gantry crane are used.

In this thesis, the gaps and drawbacks of the researchers discussed above can be resolved by considered the ideal trajectory reference and designed a non-linear controller fuzzy-PID controller to reduce the system's exposure to parametric uncertainty and disturbance.

CHAPTER THREE

MODELING

3.1 Mathematical Modeling

Since the gantry crane has two movable parts which are the rail and the trolley (cart), its motion dynamics can be mathematically modeled in two separate parts. A trolley is an electromechanical machine installed on the rail that traverses long Y-axis shown in Figure 3. The trolley carries a load attached to the hoist along the rail. The rail carries the trolley and moves along the X-axis. With the movement of the rail and the cart, we can reach any point in the x-y plane inside the working region of the crane [21].

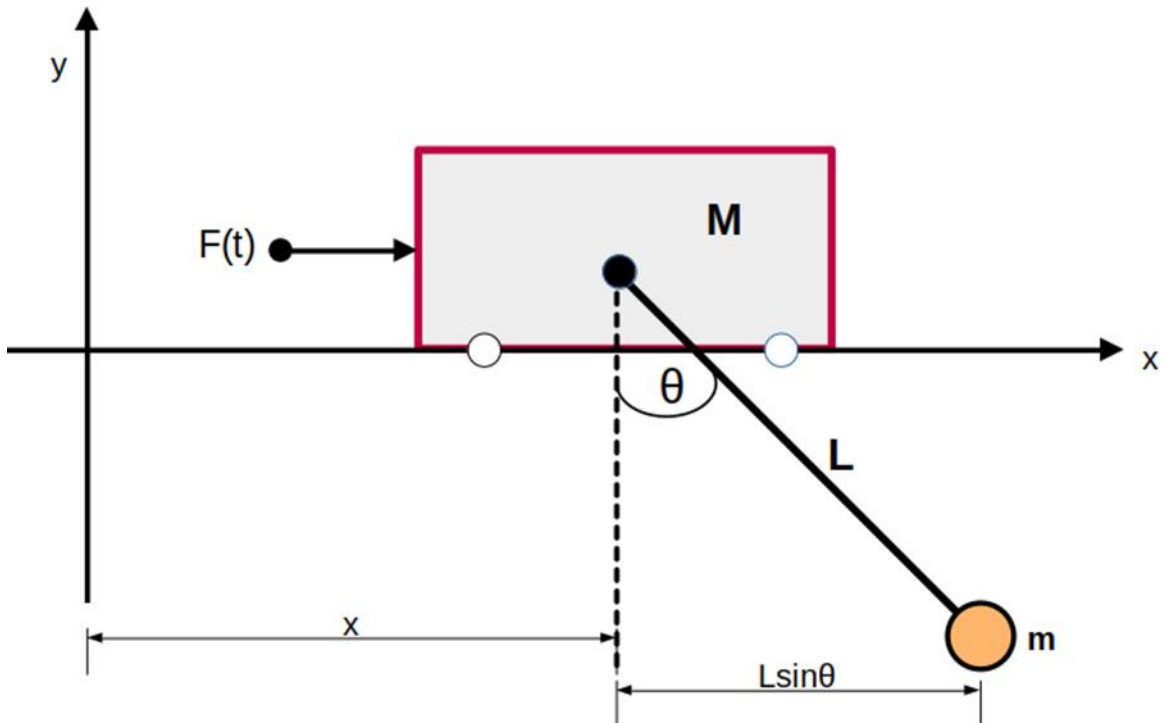


Figure 3. 1: Gantry Crane Free Body Diagram

3.1.1 Mathematical Model of the Trolley/Cart Movement

When translational force is exerted on the cart by a motor, the cart moves along the Y-axis and this creates swing dynamics of the load along the Y-axis. This swaying of the load along the Y- axis is measured as an angle which is denoted by in Figure 3.1. While modeling the cart movement; we have to describe the dynamics of the cart position denoted by y and the load swaying angle (θ). We can either use Newton's second law or the Lagrangian method to mathematically model the system [21]. The Lagrangian method is used here since it eases the complexities of the steps for gantry crane.

Lagrangian function L is described as:

$$L = T - V \quad (3.1)$$

T and V stand for the system's total kinetic and potential energy, respectively. The Lagrangian equation is provided by:

$$\frac{d}{dt} \left[\frac{dL}{du_i} \right] - \frac{dL}{du_i} = F_i \quad (3.2)$$

Where, $u_i = [x, \theta]^T$, is a generalized displacement and F_i is an external force.

T and V are used Its Lagrange equation can be applied to formalize the motion equations as in eq. (3.2). Since we have translational and rotational motion, we will use x and θ generalized displacement. The two Lagrangian equations can be written as:

$$\frac{d}{dt} \left[\frac{dL}{dx} \right] - \frac{dL}{dx} = F_x \quad (3.3)$$

$$\frac{d}{dt} \left[\frac{dL}{d\theta} \right] - \frac{dL}{d\theta} = 0 \quad (3.4)$$

The force that applies to the cart is the only external force influencing the dynamics of the swing and cart. This force which is denoted by F_x only applies on the coordinate of x which is linear displacement.

The gantry cranes kinetic energy is

$$T = \frac{1}{2} M \dot{x}^2 + \frac{1}{2} m \dot{x}_p^2 \quad (3.5)$$

Where the displacement of payload, x_p is given by

$$x_p = x + L\sin\theta \quad (3.6)$$

The differential of x_p using chain rule is given by

$$\dot{x}_p = \dot{x} + L\dot{\theta}\cos\theta \quad (3.7)$$

Substituting eq. (3.7) in to eq. (3.5) gives

$$T = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}m\dot{x}_p^2 = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}m\left[(\dot{x} + L\dot{\theta}\cos\theta)^2 + (L\dot{\theta}\sin\theta)^2\right] \quad (3.8)$$

The gantry crane potential energy is

$$V = mg\dot{y}_p \quad (3.9)$$

Where the displacement of payload, y_p given by

$$y_p = -L\cos\theta \quad (3.10)$$

The differential of y_p using chain rule is given by

$$\dot{y}_p = -L\dot{\theta}\sin\theta \quad (3.11)$$

Substituting eq. (3.11) in to eq. (3.9) gives

$$V = -mgL\dot{\theta}\sin\theta \quad (3.12)$$

Therefore the equation of motion in the x-component is given by

$$L = T - V \quad (3.13)$$

Now substituting eq. (3.8) and eq. (3.12) into eq. (3.13) gives

$$L = \frac{1}{2}(M + m)\dot{x}^2 + mL\dot{x}\dot{\theta}\cos\theta + \frac{1}{2}mL^2\dot{\theta}^2 + mgL\cos\theta \quad (3.14)$$

Plugging into fundamental langrage equation to get equation of motion in x-component of gantry crane gives

$$(M + m)\ddot{x} + mL\ddot{\theta}\cos\theta - mL\dot{\theta}^2\sin\theta = F \quad (3.15)$$

For the rotational component of the gantry crane the Lagrange equation is similar with the x-component. Therefore taking partial fraction of L with respect to θ gives the equation of motion in rotational component of gantry crane.

$$m\ddot{x}\cos\theta + mL\ddot{\theta} + mg\sin\theta = 0 \quad (3.16)$$

To make the mathematical equation suitable in the simulation of input output relationship multiplying eq. (3.16) by $\cos\theta$ and then subtract from eq. (3.15) gives

$$M\ddot{x} + m\sin^2\theta\ddot{x} - mL\dot{\theta}^2\sin\theta - mg\sin\theta\cos\theta = F \quad (3.17)$$

Therefore eq. (3.17) is the updated equation of motion gantry crane. Now, by applying small angle approximation of

$$\sin\theta \cong \theta$$

$$\cos\theta \cong 1$$

Finally, the equation of motion for the x-component in eq. (3.18) and rotational motion in eq. (3.19) are expressed.

$$\ddot{x} = \frac{mg\theta + mL\dot{\theta}^2 + F}{m + M\theta} \quad (3.18)$$

$$\ddot{\theta} = -\frac{\ddot{x} + g\theta}{l} \quad (3.19)$$

3.1.2 Mathematical Model of the Rail Movement

When a force acts on the stationary rail, the rail moves along the x-axis as shown in the coordinates in the figure 3.2 and this causes the load to sway along the x-axis. This swaying can be measured as an angle as shown in Figure 3.2.

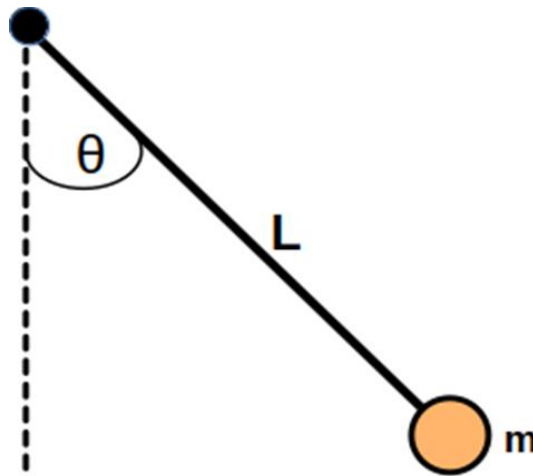


Figure 3. 2: Angular Displacement

To describe the mathematical model of the rail movement, we have driven the dynamics of the position of the rail denoted by X and the load swaying caused by it denoted by θ . Other than that, we will follow the same fashion as into driving the dynamics of the rail movement.

3.2 optimal motion-position regulator

The sway angle at the destination might be reduced to zero and an ideal position regulator where the system is driven under a trapezoidal velocity-time curve, as shown in Figure 3.3.

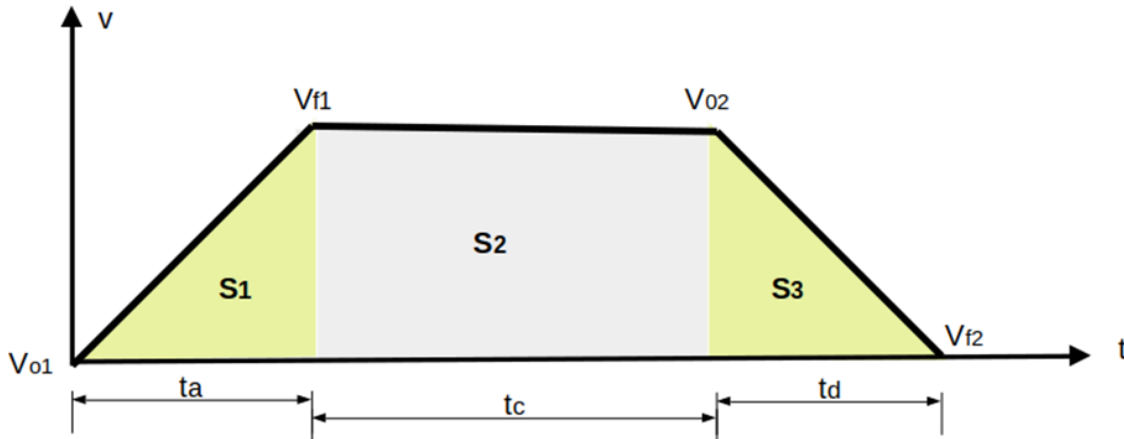


Figure 3. 3: Trapezoidal velocity plan

3.2.1 Trajectory position

This trajectory position's primary goal is to design the best trolley trajectory reference and girder trajectory reference possible given the available physical constraints.

Here, the Trapezoidal velocity-time curve depicted in Figure 3.3 is used to build an ideal desirable trolley trajectory reference and girder trajectory references. Additionally, the desired load-hoisting trajectory reference can be created in the same way if all of the loads that need to be transported from one location to the desired location have the same shapes. However, in actual use, not all loads that need to be transported have the same shapes. The controller is then provided the appropriate trajectory reference that was generated for all motion position directions. The controller follows the desired reference.

It is assumed that the acceleration and deceleration times are equal that is, i.e. $t_a = t_d$, and the entire displacement is defined as:

$$S = S_1 + S_2 + S_3 \quad (3.20)$$

While S 1 is the area covered during the time zone of acceleration, S 2 is the area covered during the time zone of constant speed, and S 3 is the area covered during the time zone of deceleration.

In the starting condition, all of the sway-angle, velocity, applied force, and acceleration as zeros, (system was at equilibrium). i.e. $\theta_x = 0$, $v_i = 0$, $a_i = 0$ and $F_i = 0$ as the initial set of constrains these parameters can be used.

3.2.1 The Displacement Equation

The region covered by the trapezoid time curve in Figure 3.3 represents the motion's displacement of the trolley and rail. We have:-

$$\begin{aligned}
 S &= \frac{1}{2} a_1 t_a^2 + v_{f_1} t_c + \frac{1}{2} a_2 t_d^2 \\
 &= \frac{1}{2} v_{f_1} t_a + v_{f_1} t_c + \frac{1}{2} v_{f_1} t_d \\
 &= \frac{1}{2} (v_0 + at_a) t_a + (v_0 + at_a) t_c + \frac{1}{2} (v_0 + at_a) t_d \\
 &= \frac{1}{2} at_a^2 + at_a t_c + \frac{1}{2} at_d^2
 \end{aligned} \tag{3.21}$$

Since $t_a = t_d$

$$S = at_a^2 + at_a t_c$$

3.2.3 Time Optimization

The load's final destination is understood to be predetermined. Therefore, depending on how quickly the cart moves, the load will arrive faster or slower. Since the load's placement is known, we can experiment with the speed and duration to minimize or maximize the sway angle. Additionally, it is evident that the sway angle of the gantry crane would increase with increasing velocity. In comparison, the sway angle will be smaller at slower speeds.

The distance is also a quadratic function of time interval based on the trapezoidal curve, as seen in eq.(3.22), where the cart's acceleration, its acceleration or deceleration time, and its constant speed time. Therefore, the solution to eq. (3.22) for time is known as:

$$t_a = \frac{-at_c \pm \frac{1}{2} \sqrt{(at_c)^2 - 4a(-s)}}{2a} \tag{3.22}$$

$$= -\frac{1}{2}t_c \pm \frac{1}{2}\sqrt{t_c^2 + \frac{4s}{a}}$$

This time equation allows us to create time limits.

$$ta, tc \geq 0$$

$$a, s \geq 0$$

$$\sqrt{t_c^2 + \frac{4s}{a}} \geq 0$$

$$t_c^2 + \frac{4s}{a} \geq 0$$

$$\sqrt{t_c^2 + \frac{4s}{a}} \geq t_c$$

$$t_c^2 > \frac{4s}{a}$$

The maximum permitted displacement; velocity, acceleration, and amount of time for moving the load are all subject to restrictions on the planned optimal motion planning.

$$s(t) = \begin{cases} \frac{v_{f_1}}{2t_a} t_1^2, & 0 \leq t_1 \leq t_a \\ \frac{v_{f_1}}{2} t_a + a_{cx} v_{f_1}, & 0 \leq t_2 \leq t_c \\ s - \frac{v_{f_1}}{2} t_a - v_{f_1} t_3 + \frac{v_{f_1}}{2t_a} t_3^2, & 0 \leq t_3 \leq t_d \end{cases} \quad (3.23)$$

$$v(t) = \begin{cases} v_0 + \frac{v_{f_1}}{t_a} t_a, & 0 \leq t_1 \leq t_a \\ v_{f_1}, & 0 \leq t_2 \leq t_c \\ v_0 + v_{f_1} - \frac{v_{f_1}}{t_d} t_3, & 0 \leq t_3 \leq t_d \end{cases} \quad (3.24)$$

$$a_{cx}(t) = \begin{cases} \frac{v_{f_1}}{t_a}, & 0 \leq t_1 \leq t_a \\ 0, & 0 \leq t_2 \leq t_c \\ -\frac{v_{f_1}}{t_d}, & 0 \leq t_3 \leq t_d \end{cases} \quad (3.25)$$

Everywhere, v_0 is the trolley's initial velocity and v_{f_1} is the final velocity of trolley known by

$$v_{f_1} = t_a \times a_{cxmax} \quad (3.26)$$

Everywhere, t_a is acceleration time, a_{cxmax} is the trolley's maximum acceleration the direction of x-axis, y-axis and along to diagonal. When we substitute eq. (3.26) in to eq. (3.23), eq. (3.24), and eq. (3.25), moreover assuming that the accelerating time is equivalent to the decelerating time. Then eq. (3.23), eq. (3.24), and eq. (3.25) are reduced to

$$s(t) = \begin{cases} \frac{a_{cx}}{2} t_1^2, & 0 \leq t_1 \leq t_a \\ \frac{a_{cx}}{2} t_a^2 + a_{cx} t_a t_2, & 0 \leq t_2 \leq t_c \\ s - \frac{a_{cx}}{2} t_a^2 - a_{cx} t_a t_3 + \frac{a_{cx}}{2t_a} t_3^2, & 0 \leq t_3 \leq t_d \end{cases} \quad (3.27)$$

$$v(t) = \begin{cases} v_0 + a_{cxmax} t_1, & 0 \leq t_1 \leq t_a \\ a_{cxmax}, & 0 \leq t_2 \leq t_c \\ v_0 + a_{cxmax} - a_{cxmax} t_3, & 0 \leq t_3 \leq t_d \end{cases} \quad (3.28)$$

$$a_{cx}(t) = \begin{cases} a_{cxmax}, & 0 \leq t_1 \leq t_a \\ 0, & 0 \leq t_2 \leq t_c \\ -a_{cxmax}, & 0 \leq t_3 \leq t_d \end{cases} \quad (3.29)$$

CHAPTER FOUR

CONTROLLER DESIGN

In this thesis both classical PID controller and nonlinear controllers of fuzzy logic controller and fuzzy-tuned PID controller are discussed in detail in the sections of this chapter that follow.

4.1 PID Controller

The most popular type of feedback is the PID controller and became the standard tool when process control emerged in the 1940s [22]. More than 95% of the control loops used in process control today is PID loops, with PI control making up the majority. Today, PID controllers are employed in every industry that employs control.

The controllers come in a variety of shapes and sizes. Each year, a hundred thousand standalone systems in boxes for one or a few loops are produced. PID controller is a crucial element of control system a dispersed. Additionally, the controllers are integrated into numerous special purpose control systems. To create the complex automation systems used in manufacturing, transportation, and energy production, PID control is frequently integrated with other elements including logic gates, sequential functions, selectors, and basic function blocks. Model predictive control (MPC), like other sophisticated control systems, is hierarchically organized. PID control is used at the lowest level; the multi-variable controller gives the set points to the controllers at the lower level [23].

The PID controller (proportional plus integral plus derivative) continuously calculates the difference between a desired set point and a measured process variable, which are errors of the supplied system. The PID (Proportional plus integral plus derivative) controller attempts to minimize the error value over time by adjustment of a control variable, such as the position of a control valve, a damper, or the power supplied to a heating element, to a new value determined by a weighted sum [24].

$$u(t) = K_p e(t) + K_I \int_0^t e(\tau) d\tau + K_D \frac{d}{dt} e(t) \quad (4.1)$$

Where K_p , K_I , and K_D , which are all positive numbers, stand for the coefficients for the respective proportional, integral, and derivative terms (sometimes denoted P, I, and D).

4.1.1 Proportional Controller (P)

The control action in proportional control is equivalent to the error at any given time as a constant, which means it accounts for the error's current values. Since a proportional controller needs a non-zero error to drive it, it typically functions with a so-called steady state error. Inverse relationships exist between the steady-state error (SSE) and the proportional gain as well as the process gain. Steady-state error may be mitigated its value by adding a compensating bias term to the set point and/or outcome of the system, or dynamically revised by including another word. The proportion gain is provided by:-

$$p_{out} = K_P \quad (4.2)$$

A proportional controller gain (K_P) will result in a reduced rise time and a reduced steady-state error, but never be eliminated.

P controller (proportional controller) forms the control deviation on the basis of the set point value and actual value, and increases this deviation with the proportional coefficient K_P [22]. In some processes, P control will generate oscillations regardless of the gain parameter's value and P control is unable to provide us with a nonzero value of the control at zero error for some types of processes.

4.1.2 Integral Controller (I)

We can allow the control to be determined by the total of the error's past values, scaled by some gain. The sum is an integral that interprets previous error values in continuous time. The magnitude and duration of the error are proportionally correlated with the contribution of the integral controller term. The entire instantaneous error over time, which is calculated as the integral controller term in a proportional plus integral plus derivative (PID) controller, yields the total cumulative offset that should have been corrected previously. The integral gain (K_I) of the integral controller is multiplied by the entire accumulated error and added to the system's controller output. The integral word of the controller is provided by:-

$$I_{out} = K_I \int_0^{\tau} e(\tau) d\tau \quad (4.3)$$

The transient response could be worsened by an integral control gain (K_I) of the controller, but it have the effects of eliminating the steady-state error. Since the integral term responds to

accumulated errors from the past, it can cause the present value to overshoot the set point value [24].

I controllers (integral-action controllers) form the areas that are enclosed over time between the control deviation and time axis. Generally speaking the controller offers the advantage that it eliminates the control deviation. However, its slow response is a disadvantage [22].

4.1.3 Derivative Controller (D)

Interprets the errors are possible future values in light of its current rate of change. Finding the error's slope over time and multiplying this rate of change by the derivative gain yields the derivative of the process error. The derivative gain is the size of the derivative term's contribution to the total control action, K_D [25]. The derivative word is provided by:-

$$D_{out} = K_D \frac{d}{dt} e(t) \quad (4.4)$$

A derivative control gain (K_D) will have the effect of increasing the stability of the system, reducing the overshoot, and improving the transient response.

Derivative action forecasts system behavior in the future, enhancing both the system's overall stability and the amount of time needed for it to settle. Because an ideal derivative controller is not causal, implementations of PID controllers include an additional filter, a low pass filtering for the derivative term, to limit the higher value of the gain and noise [25].

The common feedback control system is the proportional plus integral plus derivative (PID) controller. Through the use of integral action and derivative action, the proportional plus integral plus derivative (PID) controller is able to remove steady-state offsets. PID controllers are sufficient for many control problems, particularly when process dynamics are benign and the performance requirements are modest. PID controllers are found in large numbers in all industries [26].

Figure 4.1: demonstrate the PID controller technique that serves as a positional algorithm [27].

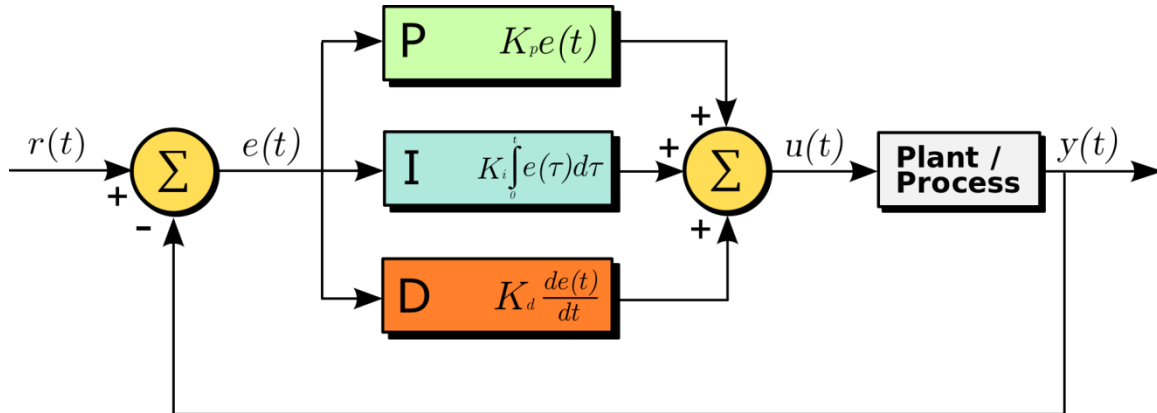


Figure 4. 1: Structure of a Proportional plus Integral plus Derivative (PID) Controller [27]

While proportional plus integral plus derivative controllers are applicable to many control problems of the real world, and often perform satisfactorily without any improvements, they can perform poorly in different applications, and do not provide optimal control due to the non-linearity of the real world problems. The feedback control system in a proportional plus integral plus derivative (PID) controller faces a number of fundamental difficulties, including the lack of direct process information, constant parameters, and reactive performance.

PID controllers can perform poorly when used alone when PID loop gains must be reduced to prevent overshoot, oscillation, or searching around the control set point value. PID Controller is a feedback controller that uses the integral, derivative, and weighted sum of the error to control the plant [25]. The reference signal and output signal of the entire system serve as the input and output signals for the PID controller. Respectively, PID controllers have the disadvantage of being unable to respond to sudden (random) changes in the error signal. Because it can only calculate the error signal's instantaneous value without taking into account how the error changes as it rises and falls.

The most significant enhancement is the addition of feed-forward control using system information and limiting the use of the PID to error control. PIDs may also be improved by changing the parameters (either adaptively changing them based on performance or gain scheduling in various use cases), increasing sampling rate, accuracy, precision, and low-pass filtering, if necessary, or cascading numerous PID controllers.

Table 4. 1: Parameter of PID controller for sway and trolley

Parameters	Value
K_P	207.65
K_I	61.06
K_D	122.81

4.2 Fuzzy Logic Controller

Fuzzy control provides a formal methodology for representing, manipulating, and implementing a human's heuristic knowledge about how to control a system.

Physical systems that have difficult of system structure, non-linearity, uncertainty, randomness, etc. are very challenging to model with a precise and exact mathematical formula or equation. Fuzzy logic controllers have the advantages of handling non-linearity, operating with incorrect inputs, not requiring an accurate mathematical model, and controlling complex systems.

In fuzzy logic, a set of linguistic rules selected by the system determines the fundamental control. The system does not need to be mathematically modeled because numerical variables are transformed into linguistic variables. The reference signals are fuzzyfied, and membership functions in fuzzy set notations are used to express them. The well-defined "IF" and "THEN" rules generate output (actuating) signals, which are then defuzzified to provide crisp values [28].

Artificial intelligence's fuzzy logic field focuses on the reasoning techniques that robots employ to mimic human thought. In situations when process data cannot be expressed in binary form, these techniques are employed. For instance, "the air feels hot" and "he is young" are not mutually exclusive. It analyzes confusing statements so that they make logical sense.

Fuzzy logic, as a result, needs expertise. The fuzzy system stores this information, which was supplied by an expert who understands the process or machine. Fuzzy logic has existed since Aristotle's discovery of the excluded middle principle. Aristotle claimed that in the field of logical reasoning, the middle ground is abandoned; statements can only be true or false.

When FLCs were advanced, its discrete logic was based on dated ideas. As a result, all other values were disregarded, and only one set of inputs and outputs could exist (i.e., ON or OFF). Items can belong to many sets according to fuzzy logic in FLCs, to belong to the concept of the excluded middle.

One of the first fuzzy systems was created in 1975 by Professor Ebrahim Mamdani of London University to manage a steam engine and boiler combination.

The four steps of the Mamdani style fuzzy inference procedure, the most widely used fuzzy inference method, are as follows:

- Fuzzification,
- Rule evaluation;
- Aggregation of the rule outputs,
- Defuzzification.

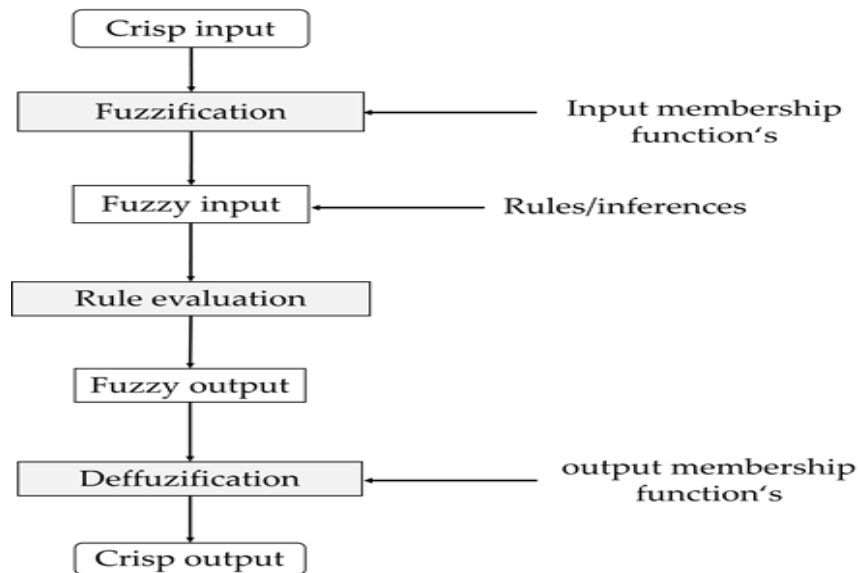


Figure 4. 2: Operation of Fuzzy System [30]

A fuzzy logic controller performs three the main tasks. Fuzzification, fuzzy inference (processing) and Deffuzification are shown in Figure 8.

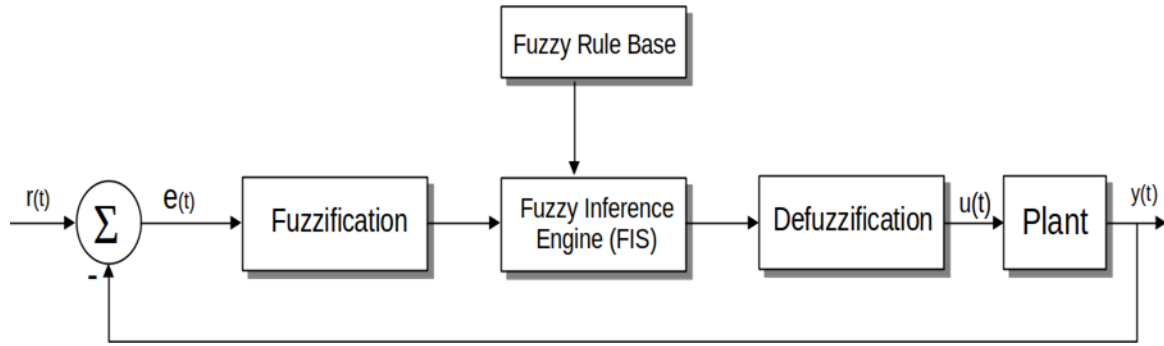


Figure 4. 3: Fuzzy Logic Controller [30]

4.2.1 Input Parameters

Two variables, output of the system, the input signal and error rate i.e. the fuzzy logic controller takes the rate of error change as input. Error (e) and error rate (Δe)

$$\Delta e = e(n) - e(n - 1) \quad (4.5)$$

4.2.2 Fuzzification

The Fuzzification module converts the physical values of the current process signal, the error signal that is fed into the fuzzy logic controller, into a normalized fuzzy subset made up of an interval for the range of input values and an associated membership function that specifies the degrees of confidence in the input's membership in this range.

The Linguistic variables like negative big (NB), negative medium (NM), negative small (NS), zero (Z), positive small (PS), positive medium (PM), and positive big (PB), which are represented by triangular membership functions, are used to specify the error and the rate of change of the error. These functions have been chosen in order to satisfy the fuzzy controller's output requirements. Seven linguistic variables, including NB, NM, NS, Z, PS, PM, and PB, are also used to determine the output. Their membership functions are shown in Figure 4.6.

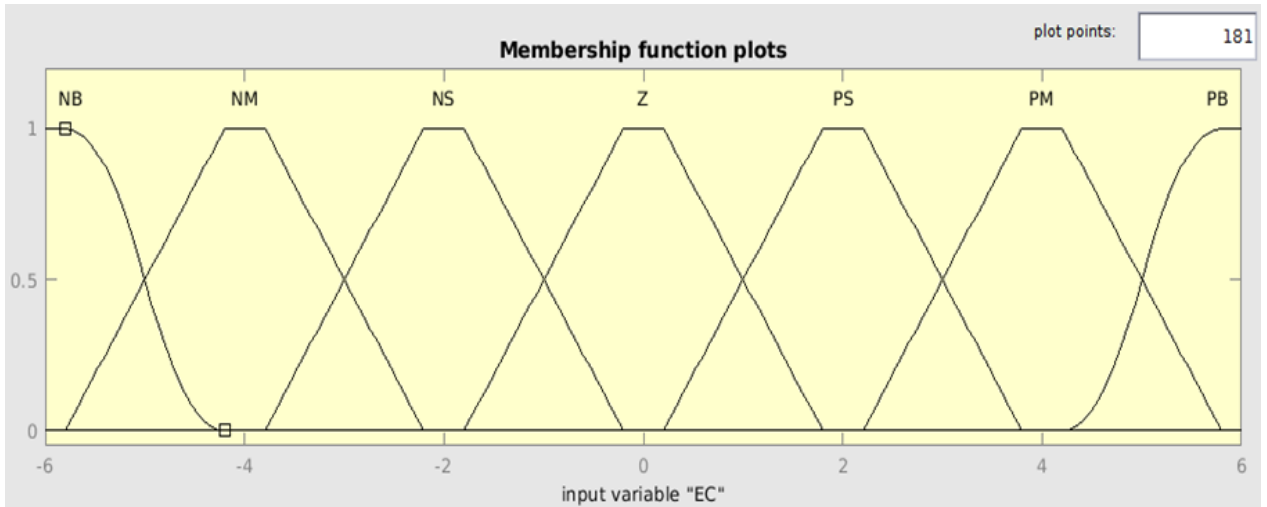


Figure 4. 4: Error Membership Function

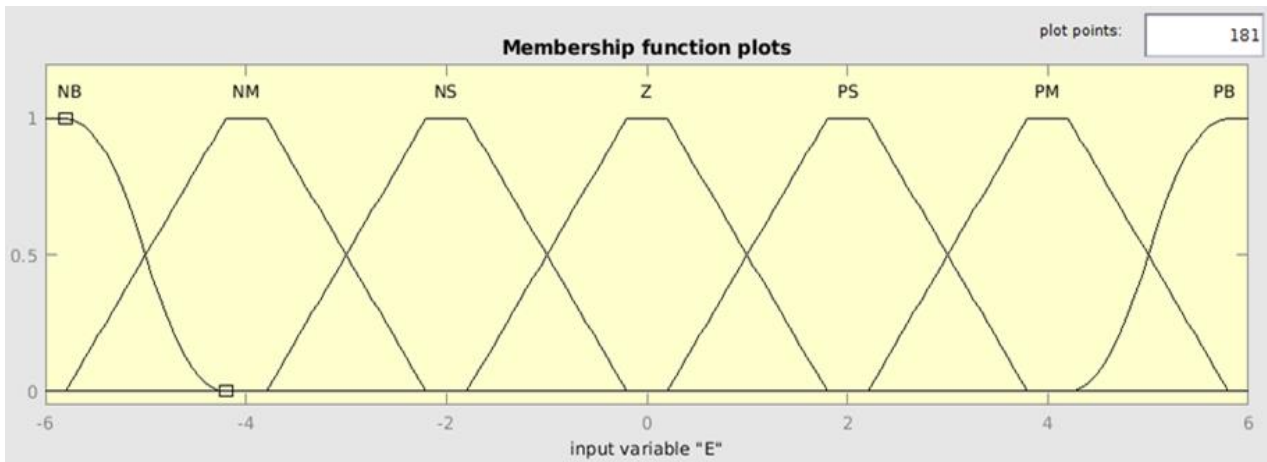


Figure 4. 5: Derivative of error membership function

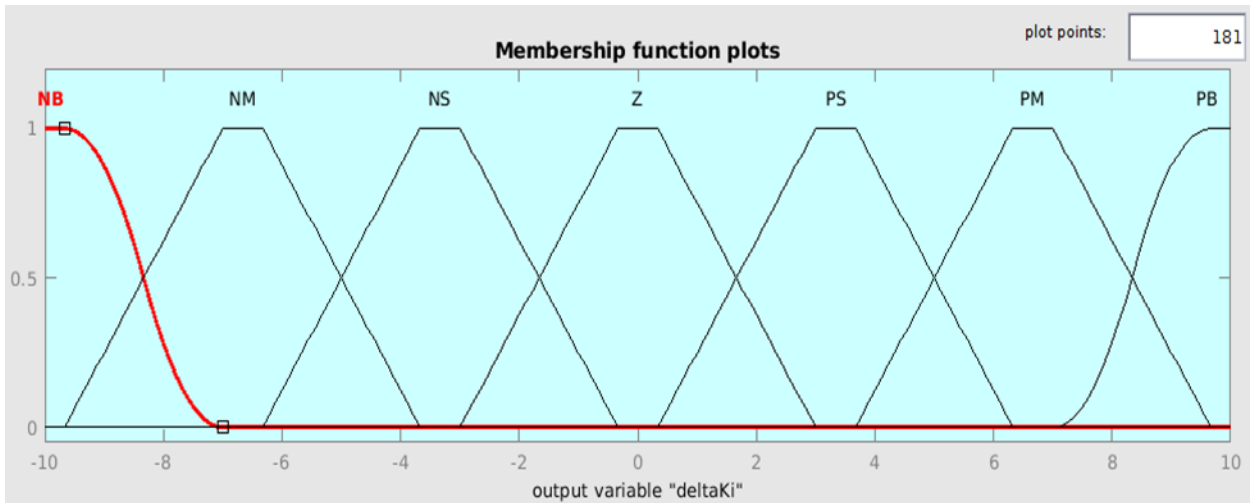


Figure 4. 6: Output Membership Function

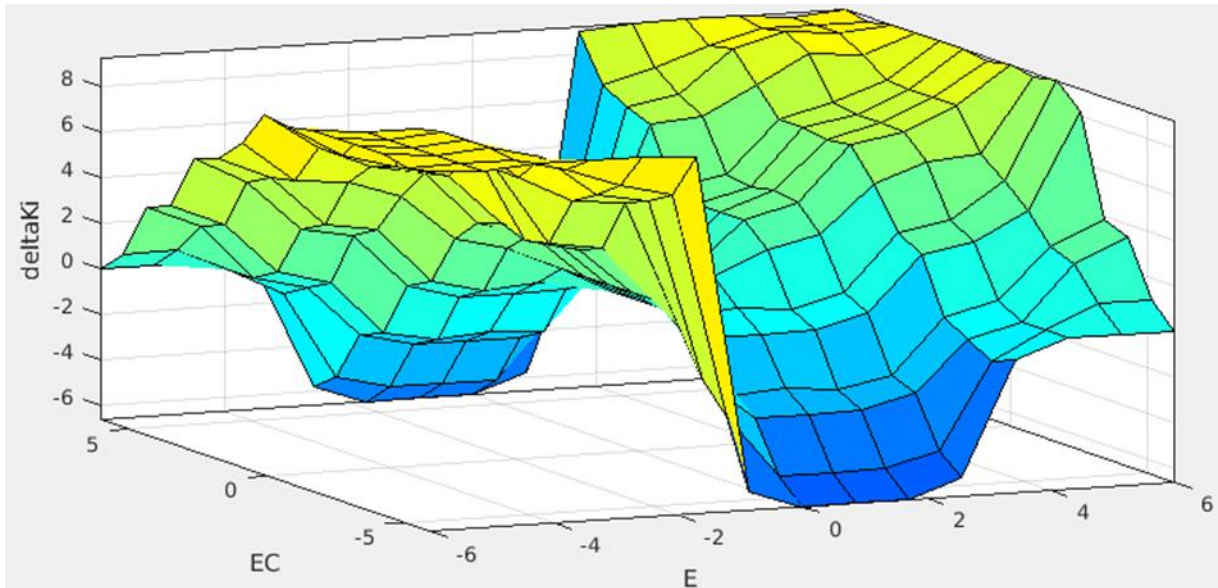


Figure 4. 7: Surface of Fuzzy rules of input/output Membership Function

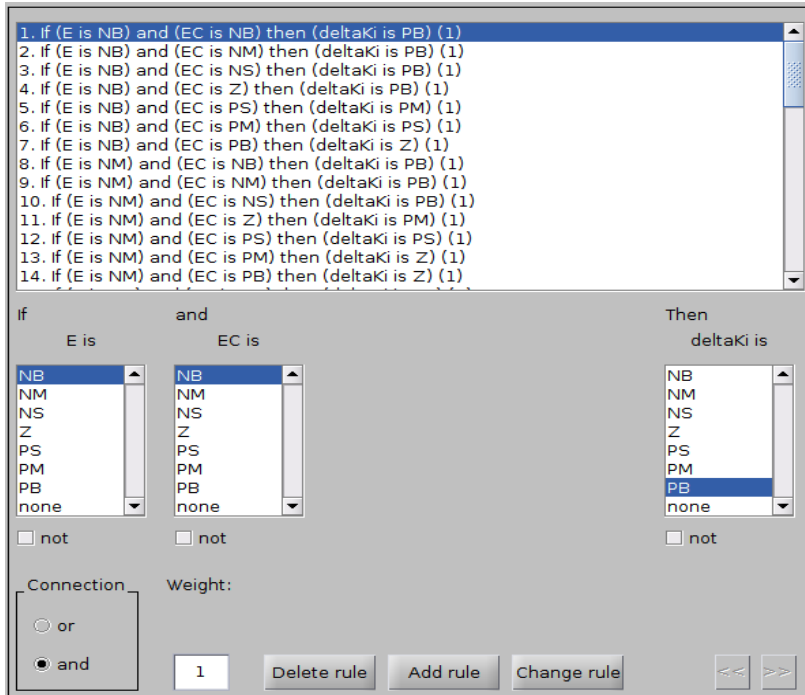


Figure 4. 8: Rule Base

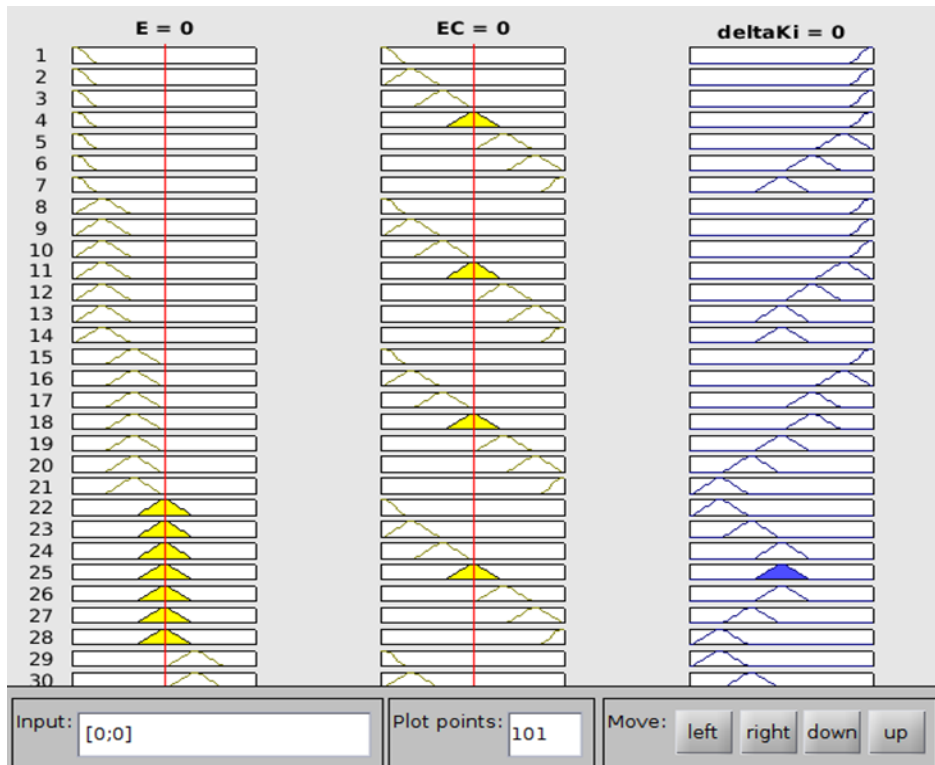


Figure 4. 9: Rule Viewer

4.2.3 Decision Making

Mamdani's inference approach is used to implement fuzzy processes. As a result of Mamdani's fuzzy inference system's suitability for human input, intuitive, and can quickly determine how its inputs and outputs relate to one another. In this thesis, triangular membership function is used because of simplicity. The “IF . . . THEN . . . “principles characterize a fuzzy inference system (FIS) by associating the output to the inputs.

By analyzing all the rules, fuzzy sets and fuzzy logic operations generate the outcome. It is clear that, it is better to understand from Table 1 which has 49 rules for the seven (7) linguistic variables and the combination of rules are expressed as follows. Where E represents for error and CE for change in error.

- If E is NB AND ΔE is NL THEN output is NB
- If E is NM AND ΔE is NL THEN output is NB
- If E is NS AND ΔE is NL THEN output is NB

Table 4. 2: Rule base of the fuzzy controller

		Error						
		NB	NM	NS	Z	PS	PM	PB
Change of Error	NB	NB	NB	NB	NB	NM	NS	Z
	NM	NB	NB	NB	NM	NS	Z	PS
	NS	NB	NB	NM	NS	Z	PS	PM
	Z	NB	NM	NS	Z	PS	PM	PB
	PS	NM	NS	Z	PS	PM	PB	PB
	PM	NS	Z	PS	PM	PB	PB	PB
	PB	Z	PS	PM	PB	PB	PB	PB

4.2.4 Defuzzification

The act of changing linguistic labels represented by fuzzy sets in the controller outputs to crisp values is known as defuzzification. The last crisp output value of the fuzzy logic controller depends on the type of defuzzification approach to generate the outcome values

corresponding to each label. The input for the Defuzzification process is the aggregated output fuzzy set from the rule base of the fuzzy logic controller and the output is a single crisp number.

Following their logical addition, all of the rule outputs are examined as part of the defuzzification process, which then calculates a value that will serve as the fuzzy controller's final output. The output module receives this value from the fuzzy controller. Consequently, the controller changes the fuzzy outcome into a real-world data value during defuzzification [29]. Although there are numerous defuzzification techniques, they are all founded to mathematical procedures. The two most popular methods for defuzzification are:

Maximum Value Method: The maximum value technique used with discrete output membership functions and the final output value is calculated using the rule output that received the highest grade for the membership function.

Center of Gravity Method: The center of gravity technique, also discussed to as “calculating the centroid,” mathematically achieves the center of mass of the triggered output membership functions. This Defuzzification technique is the most widely utilized since it yields a precise result based on the weighted values of several output membership functions. Both continuous and non-continuous output membership functions can be solved using the middle of gravity technique.

In the center of gravity technique of defuzzification, the area and the centroid of each sub-area is calculated and then the summation of all these sub-areas is taken to find the defuzzified value for a discrete fuzzy set. In center of gravity method, if x^* is the defuzzified value and $\mu(x_i)$ is a membership function, the defuzzified value x^* is given by:

$$x^* = \frac{\int x\mu_A(x)dx}{\int \mu_A(x)dx} \quad (4.6)$$

4.3 Fuzzy-tuned PID Controller

The PID controller is very common in the control of gantry cranes, and better for minimizing steady-state error. However, one disadvantage of this classical controller is that it may not give the necessary control performance when there are variations in the system parameters

because it uses constant gains. The fuzzy logic controller overcomes the PID controller's drawback.

One of the nonlinear controllers that can be used to control gantry cranes is the fuzzy logic controller. This non-linear controller can deliver acceptable performance when influenced by shifting system parameters and operating conditions, as opposed to the linear PID controller. The fuzzy logic controller's function is highly helpful since it frees the system from laborious mathematical calculations and modeling.

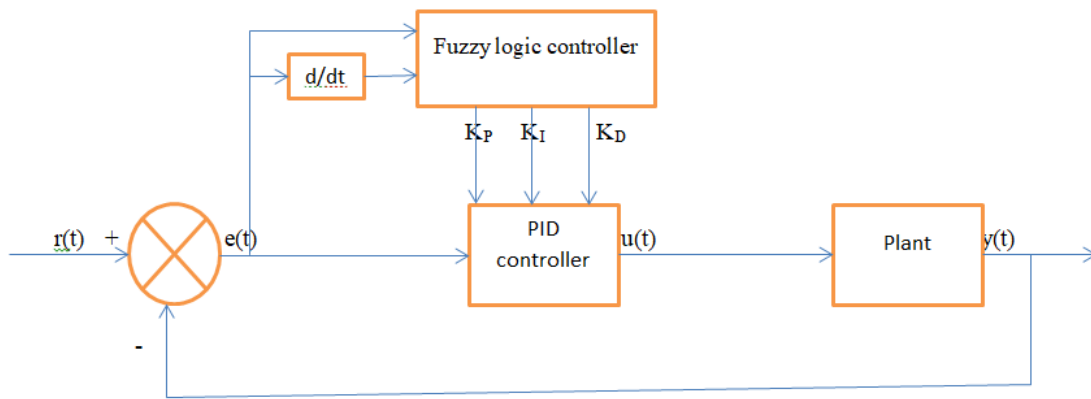


Figure 4. 10: Fuzzy-Tuned PID Controller [31]

In this thesis, a fuzzy-tuned PID controller is suggested as a solution to this issue and to enhance the performance of gantry cranes. In Fuzzy-tuned PID controller, the error and rate of change of error signals are passed through the PID controller. These signals are processed by the fuzzy logic controller and their magnitudes are brought to a range desirable for input to the designed PID controller. Fuzzy-tuned PID controller is the combination of conventional PID controller and fuzzy logic controller in which hybrid controllers are expected to improve the performance of nonlinear systems like gantry crane. Due to the nature of the gantry crane (which is non-linear) and the exact condition of the fault will happen to the system is random, the PID controller is not recommended alone. Therefore, good features of PID controller and fuzzy logic controller makes the system control strategy better.

CHAPTER FIVE

RESULTS AND DISCUSSION

5.1 Introduction

The gantry crane control systems has been discussed and analyzed in previous chapters. The simulation result of the system is presented in the upcoming sections of this chapter. The magnitude values, length, and other parameters are taken and simulated in MATLABTM.

5.2 Simulation Results

Based on the designed model and controllers on the previous chapters, the simulation results for open loop control systems, using PID controllers and Fuzzy-tuned PID controller are discussed and analyzed in the upcoming sections.

5.2.1 Simulation Results in optimal motion-position regulator

This section presents the time-optimization simulation results for the displacement, sway angle, and velocity time curve. As a result, the weight that needs to be transported arrives at its destination in an ideal amount of time with no sway. The controller design in this thesis consists of two control techniques. In the first stage, an ideal motion position regulator is created using the system input coordinates in the MATLAB workspace and acts as the systems desired reference inputs. The main purpose of this motion position regulator is to quickly track the desired displacement input trajectories and anti-sway reference trajectories while minimizing disturbance.

The signal builder of motion position trajectory used in the simulation is shown in Figure below.

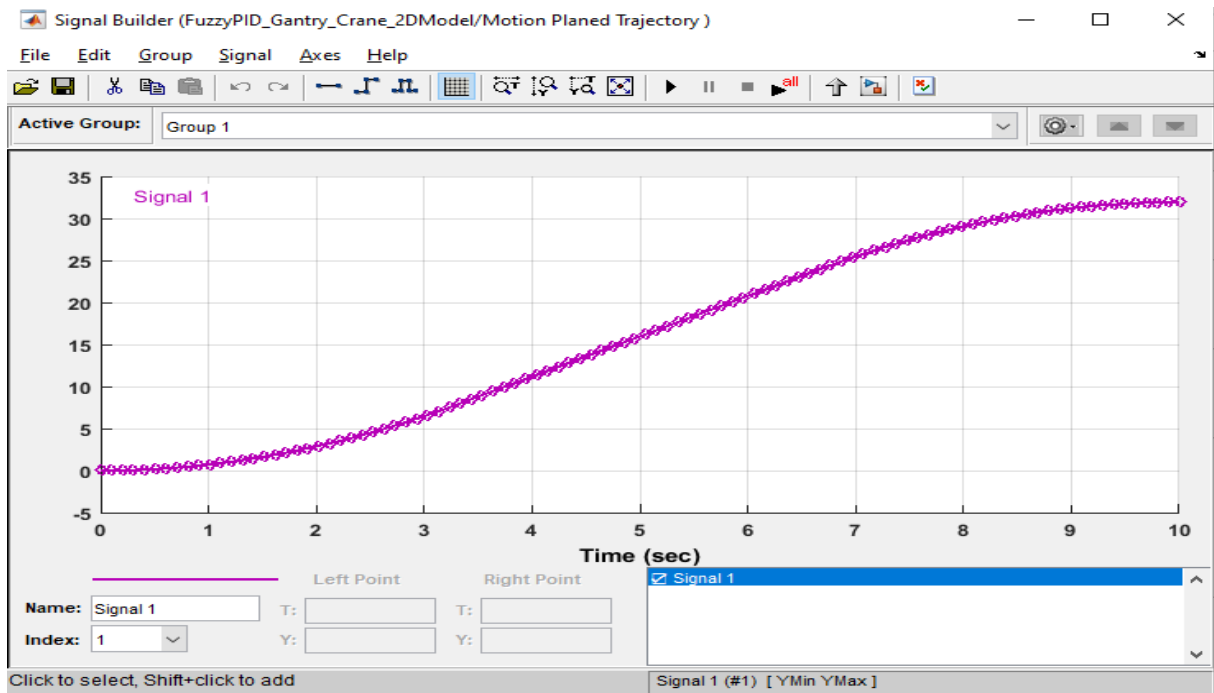


Figure 5. 1: Signal Builder of Motion Position Trajectory

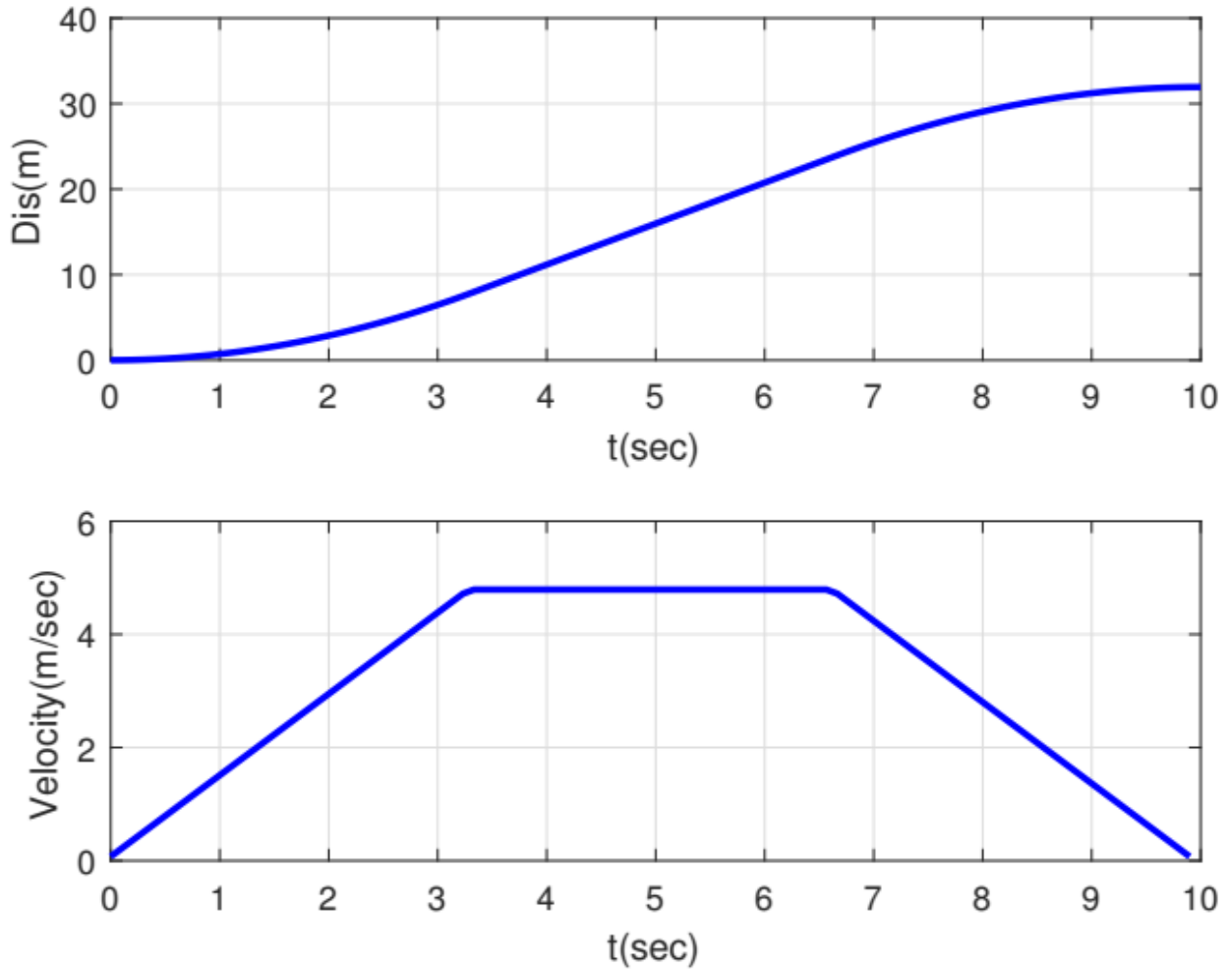


Figure 5. 2: The Motion Position Velocity Trajectory Path for Reference Signal

5.2.2 Simulation Result of Open Loop Control of Gantry Crane

The simulation result of the gantry crane without controller i.e., open loop control system of the gantry crane is depicted in figure 5.5 of the position and velocity control and its sway angle and angular velocity is also depicted in figure 5.6. From the simulation diagram for both position and sway angle control of the gantry crane is not following the motion position path. Therefore, without controller the gantry crane is unstable.

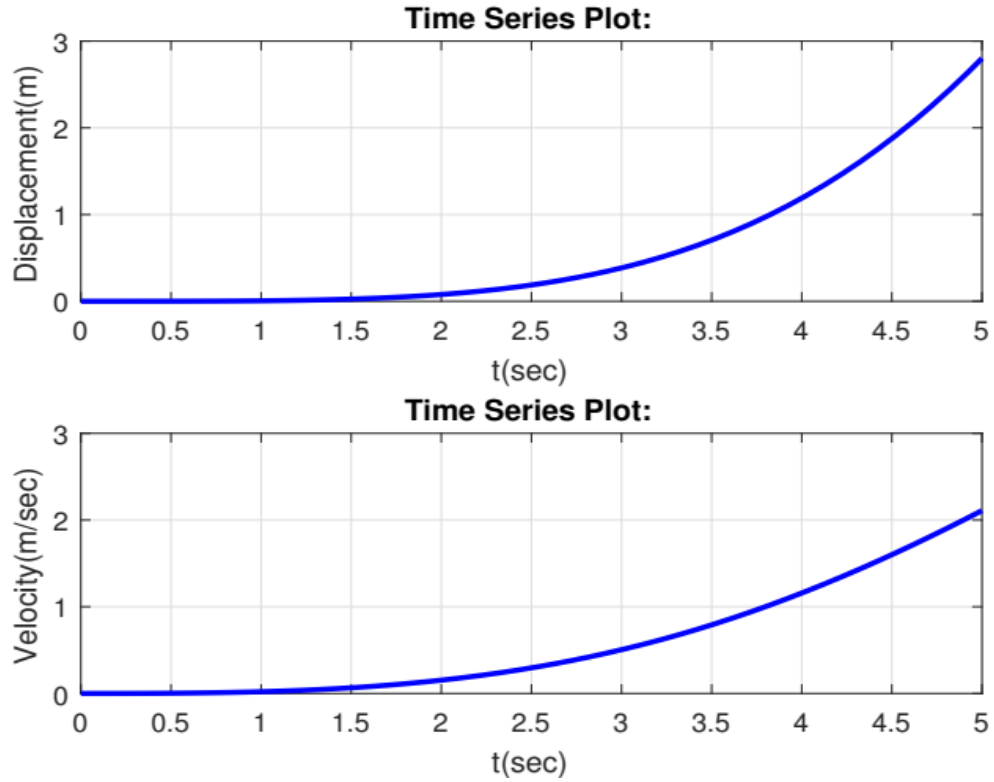


Figure 5. 5: Position of Gantry Crane without Controller

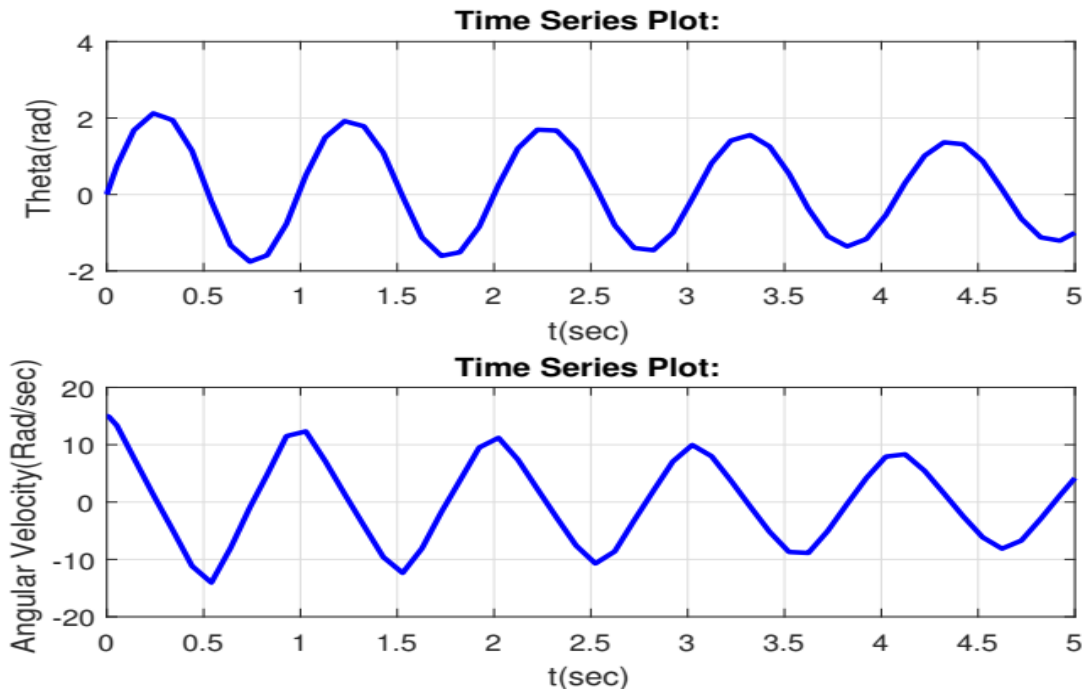


Figure 5. 6: Sway Angle Control of Gantry Crane without Controller

5.2.3 Simulation Results Using PID Controller

The tracking performance of cart velocity, cart displacement, and swaying angle are tested using a PID controller that is constructed as a benchmark in this simulation results. The motion position reference input is tracked by the PID controller that was designed.

The velocity of the cart and its payload increase linearly until it reaches the desired maximum velocity. After reaching their respective maximum velocities, they continue to move at a constant velocity for a certain time, while before beginning to linearly decrease their velocities until they approach zero, as illustrated in Figure 5.8. The ideal reference velocity for the cart and the load, as determined through two-dimensional gantry crane mathematical optimization, is shown in Figure 5.8. To following the trapezoidal curve shown in Figure 5.2: For this estimated result the system follows this reference velocity-time curves.

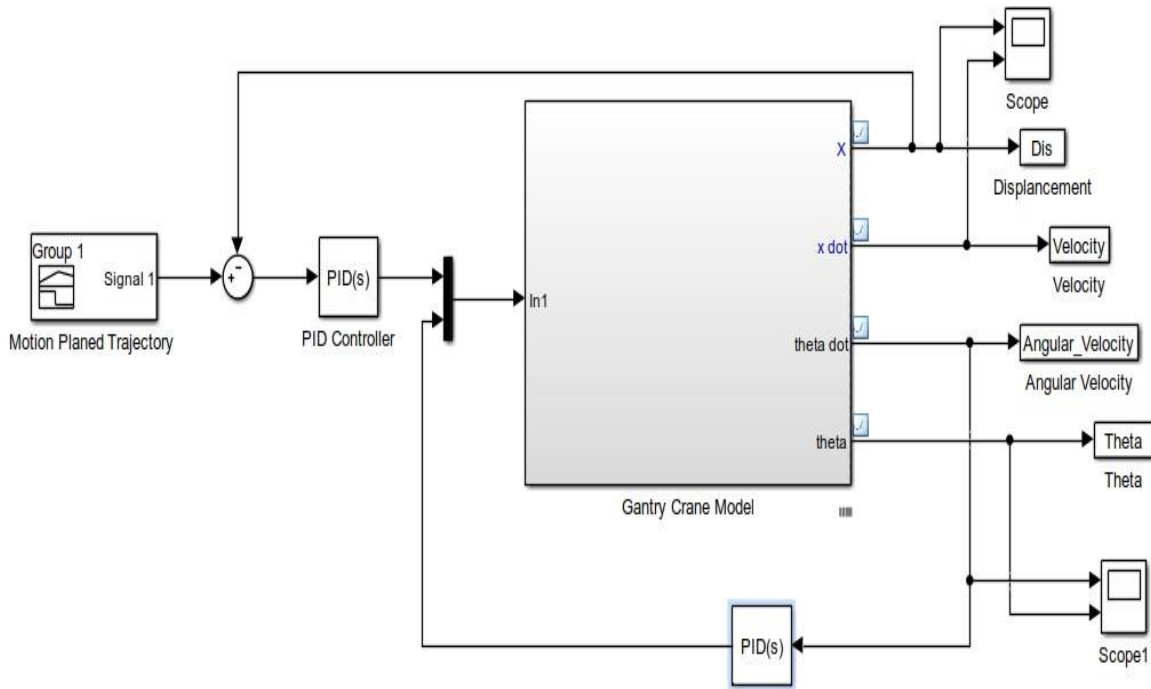


Figure 5. 7: Simulation Model of Gantry crane using PID Controller

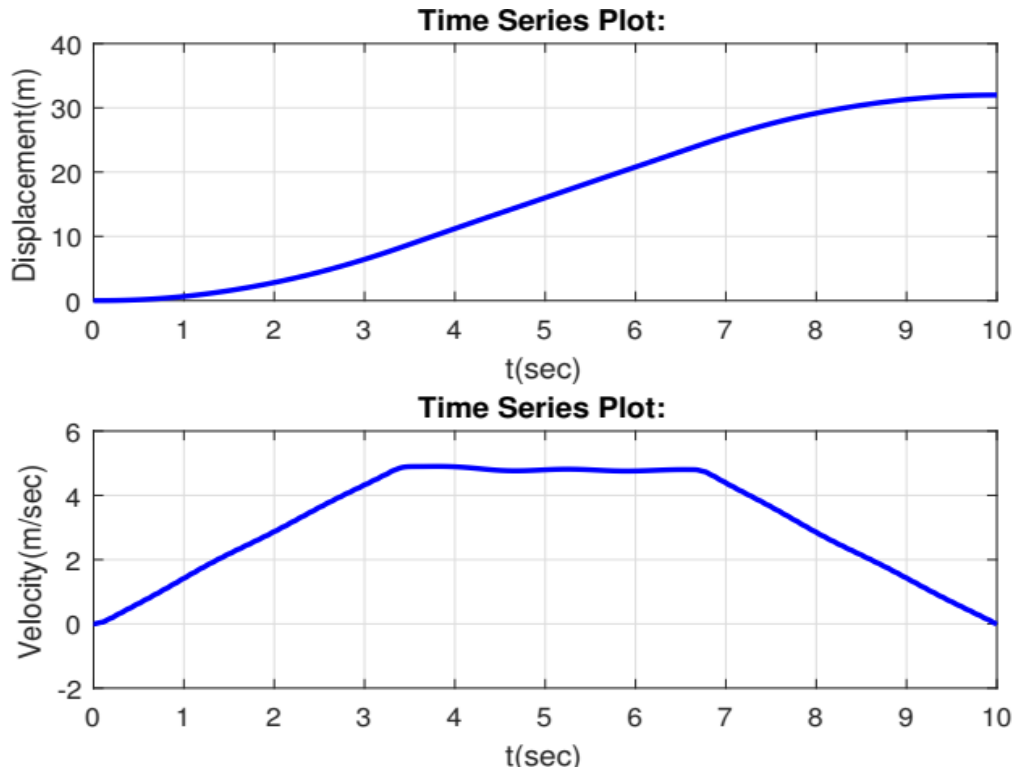


Figure 5. 8: The Trajectory path tracking of Gantry Crane PID Controller

Figure 5.9 illustrates that the PID control system's sway angle settling time takes a long time. For this cause the crane will not reach its determined location and determined time using PID control system.

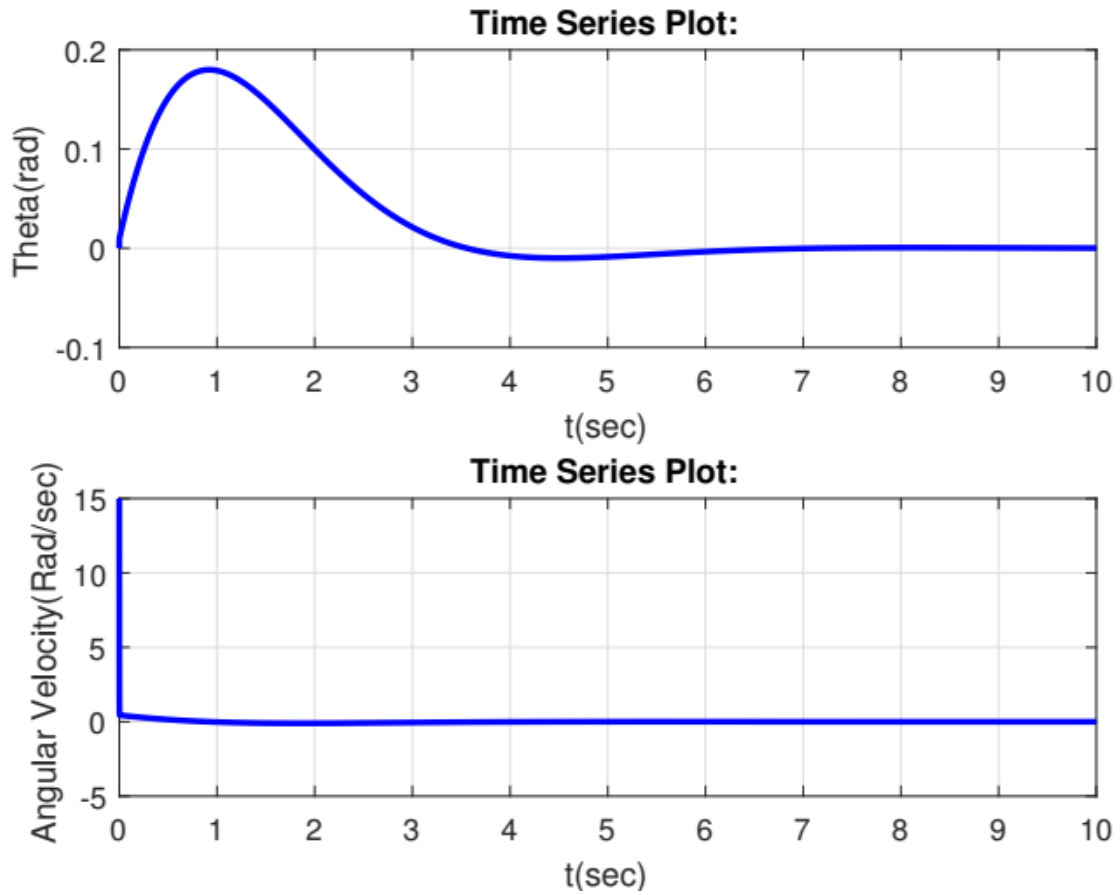


Figure 5. 9: The Sway Angle and Angular Velocity of the payload Gantry Crane using PID Controller

5.2.4 Simulation Results Using Fuzzy-tuned PID Controller

The tracking performance of cart velocity and displacement is tested using a PID controller. PID controllers are ineffective in the presence of parameter variations. In order to manage parameter uncertainty and track optimum inputs, fuzzy-tuned PID Controller is designed. The gantry crane overall model with fuzzy-tuned PID Controller is designed in MATLAB as shown in Figure 5.10 and the fuzzy-tuned PID controller is depicted in Figure 5.11.

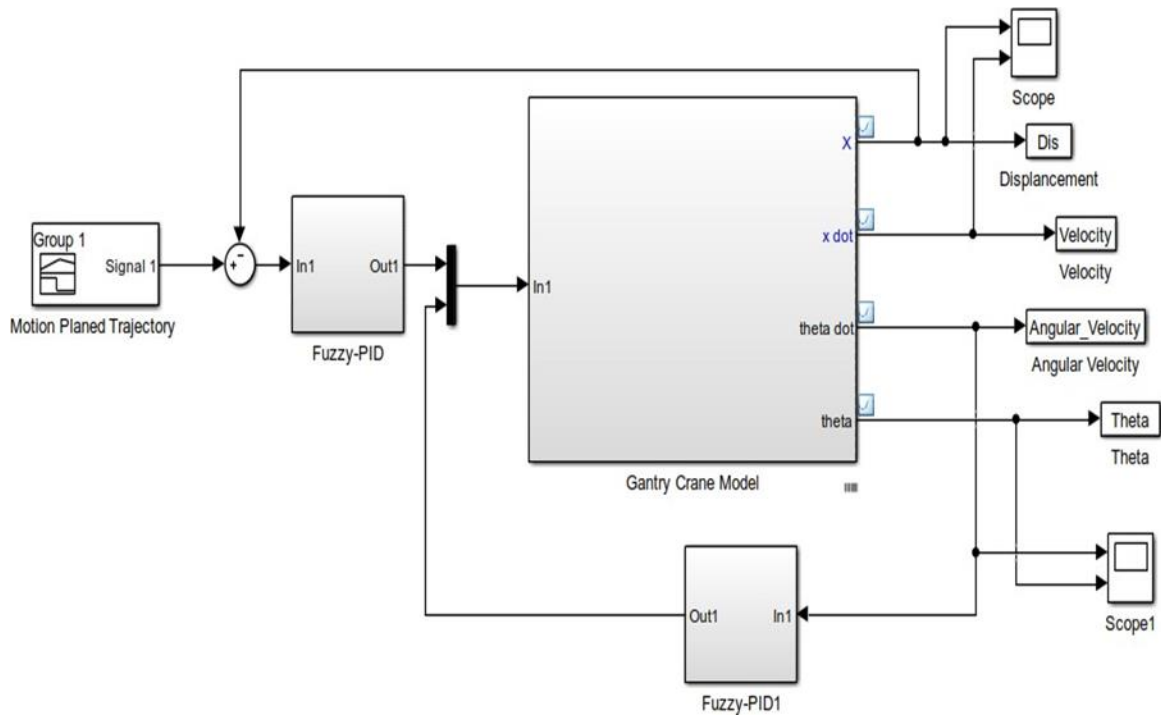


Figure 5. 10: Simulation Model of Gantry Crane using Fuzzy-Tuned PID Controller

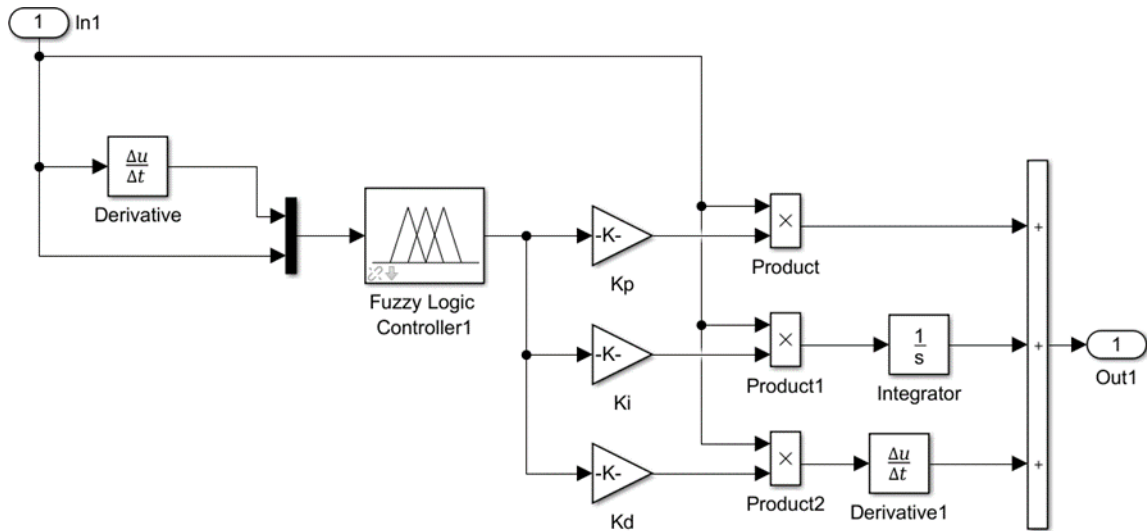


Figure 5. 11: Simulation Diagram of Fuzzy-Tuned PID Controller

Fuzzy-tuned PID controller has very small time required to settle about the sway angle compared to PID controller. This leads to the conclusion that fuzzy-tuned PID controllers perform better at tracking sway angles than PID controller methods.

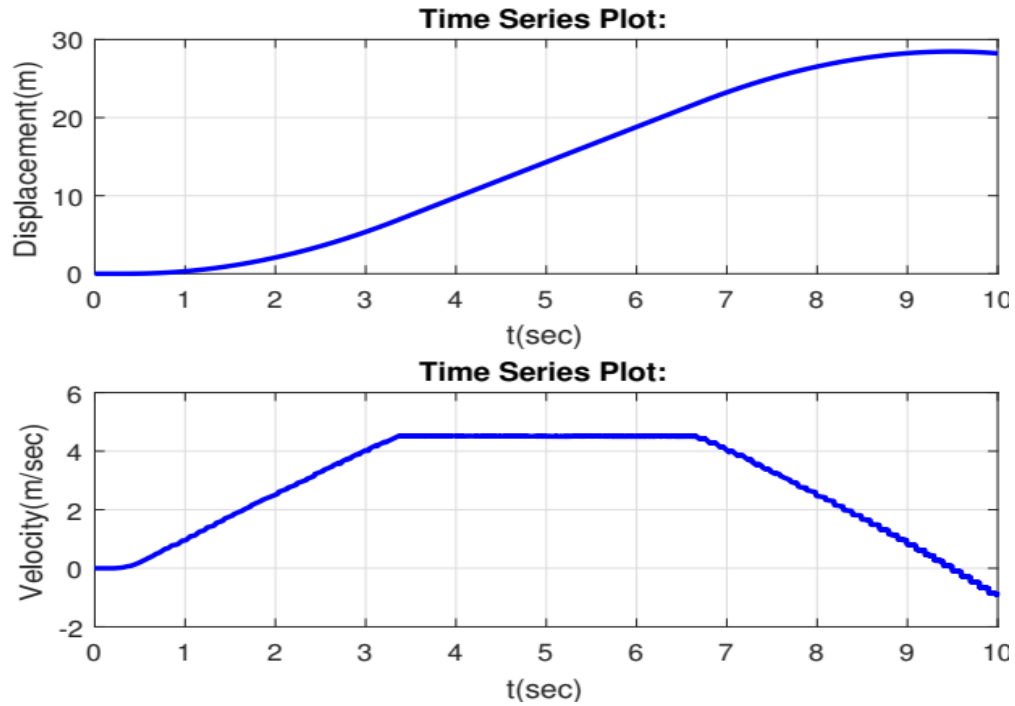


Figure 5. 12: The Trajectory path tracking of Gantry Crane using Fuzzy-Tuned PID Controller

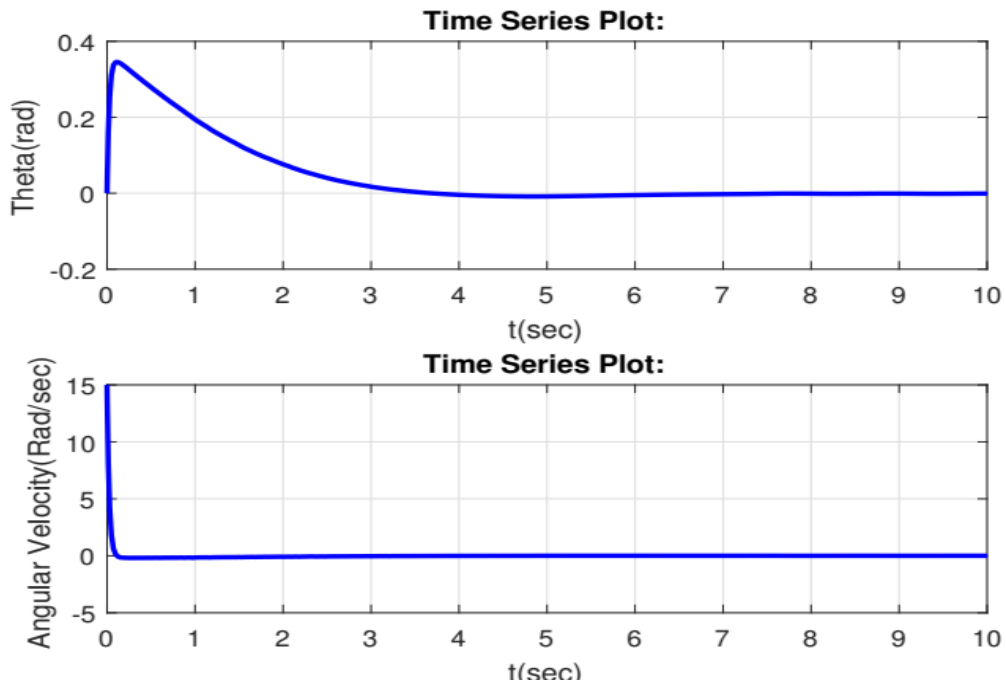


Figure 5. 13: The Sway Angle and Angular Velocity of the payload of Gantry Crane using Fuzzy-Tuned PID Controller

5.3 Discussion

The simulation results of the gantry crane control using both classical controller and fuzzy-tuned PID controller are presented in the previous sections of this chapter. From the simulation results presented above the following discussions are drawn.

As shown from figure 5.13, the position control of the gantry crane using PID controller and fuzzy-tuned PID controller are compared. According to the simulation result the PID controller is better in position control of the gantry crane with respect to the path tracking of the gantry crane.

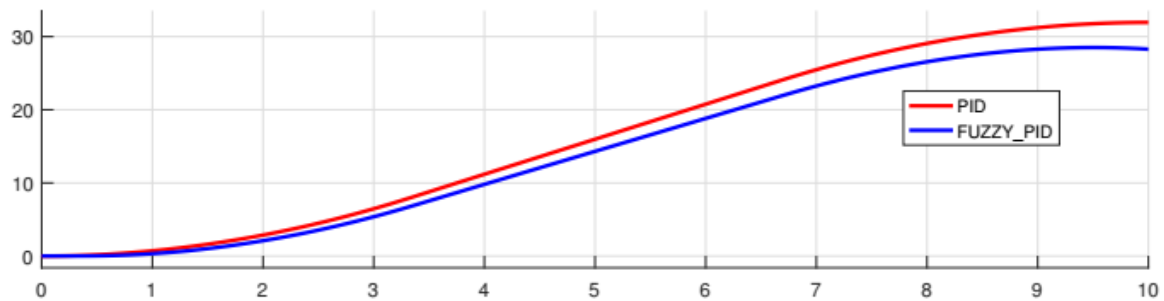


Figure 5. 14: Comparison of Position of Gantry Crane

As shown from figure 5.14, the velocity control of the gantry crane using PID controller and fuzzy-tuned PID controller are compared. According to the simulation result the both the PID controller and fuzzy-tuned PID controller are almost similar in velocity control of the gantry crane with respect to the path tracking of the gantry crane.

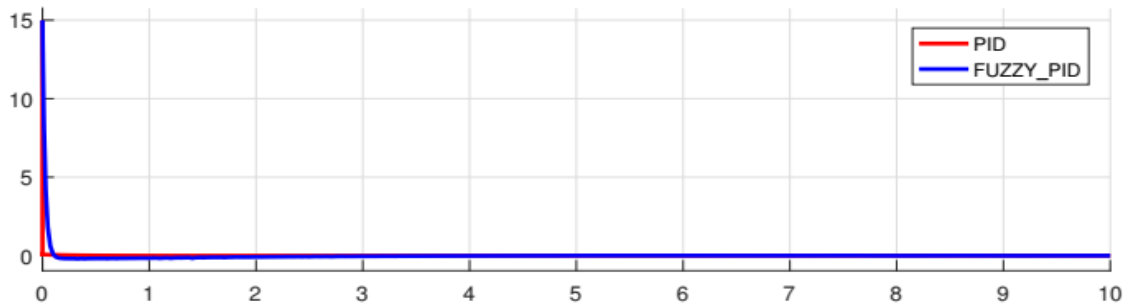


Figure 5. 15: Comparison of Velocity of Gantry Crane

As shown from figure 5.15, the sway angle control of the gantry crane using PID controller and fuzzy-tuned PID controller are compared. According to the simulation results, the fuzzy-

tuned PID controller performs better than the conventional PID controller in controlling the gantry cranes sway angle in terms of tracking its path.

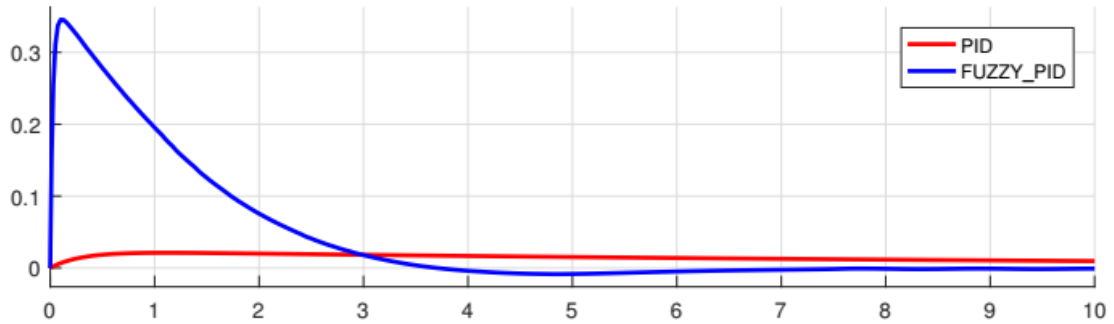


Figure 5. 16: Comparison of Sway Angle (theta) of Gantry Crane

As shown from figure 5.16, the angular velocity control of the gantry crane using PID controller and fuzzy-tuned PID controller are compared. In terms of the gantry crane's angular velocity control and path tracking, the simulation results show that the fuzzy-tuned PID controller performs significantly better than the PID controller.

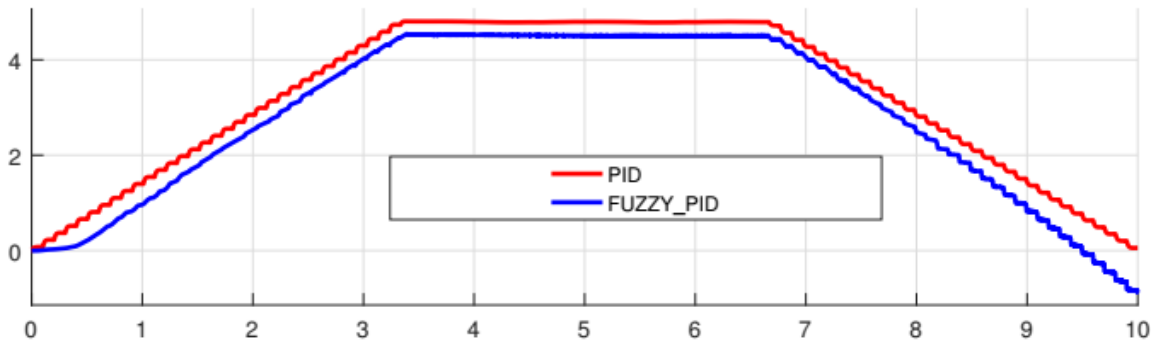


Figure 5. 17: Comparison of Angular Velocity of Gantry Crane

In general, PID controllers are better for gantry crane position and velocity control. Whereas fuzzy-tuned PID controllers are more effective than classical PID controllers for sway angle control and angular velocity control.

When a disturbance is added into the system the simulation results are exactly the same with the simulation results discussed above. And the disturbance added to the system is given in Figure 5.18.

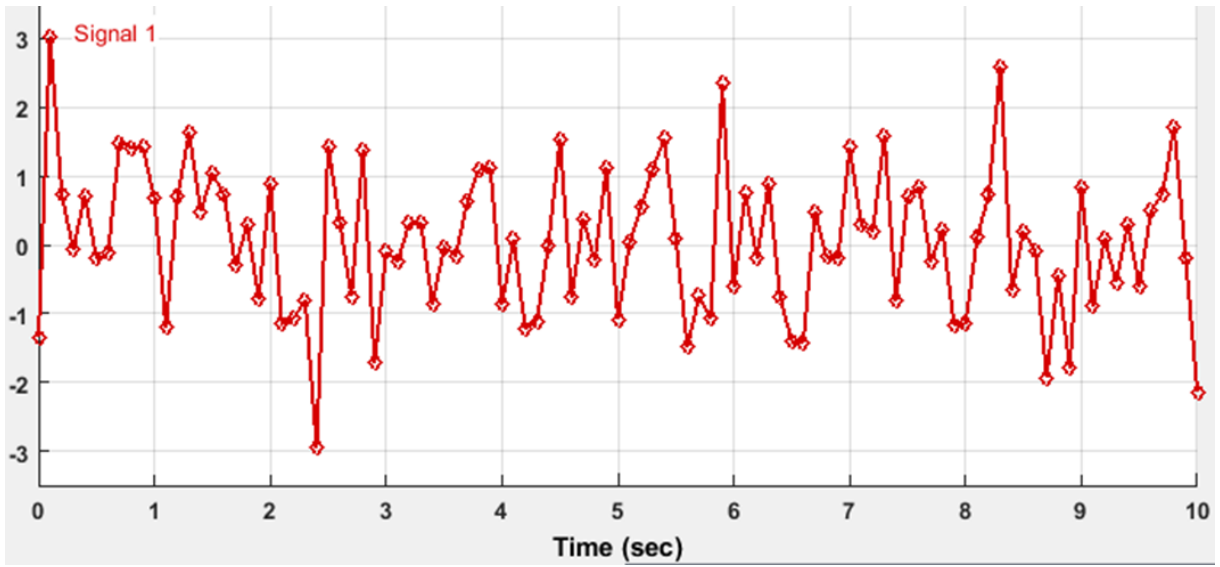


Figure 5. 18: Disturbance Added to the System Dynamics

CHAPTER SIX

CONCLUSION AND FUTURE SCOPE

6.1 Conclusion

In this thesis, the modeling and simulation of gantry crane controlled by fuzzy-tuned PID Controller have been developed. From the simulation results discussed in chapter five, the overall crane system's safety is guaranteed, and the payload's time efficiency while transportation is improved. Additionally, the trolley rapidly arrives at the desired location with low payload sway.

These goals were attained using two design stages, which included fuzzy-tuned PID design for 2D gantry cranes and systematic anti-swing optimal motion planning. In this thesis, PID and fuzzy-tuned PID controllers are designed and simulated in MATLAB. The PID controller's parameters are automatically tuned in MATLAB Simulink, and it is designed to track the ideal reference inputs.

Finally, the performance comparisons of the PID controller and fuzzy logic controller (FLC) are in order to achieve the best reference velocity tracking performance. The simulation result of fuzzy-tuned PID controller shows that better optimal sway angle tracking performance compared to PID controllers. It can be conclude that fuzzy-tuned PID logic controller has better performance on the position and sway angle control.

6.2 Future Scope

In this thesis, fuzzy-tuned PID control systems and motion planning are proposed, designed, analyzed, and simulated using MATLAB. For the future work this thesis can be done in different advanced controllers and other planning methods. Using intelligent control methods like artificial intelligence, reinforcement learning methods will be the best data training methods of gantry crane sway control and position controls. Since controlling the gantry crane with real data's will be good controlling mechanism. Not only the controller but also the dynamics of controllers also will be extended to three dimensional (3D) model of gantry crane.

REFERENCES

- [1] M. S. Mahmoud, N. M. Alyazidi, and A. M. Hassanine, "Modeling and Control of Under actuated Three-Dimensional Overhead Crane Systems," *Int. J. Robot. Autom, Technol.*, vol. 6, pp. 80–85, 2021.
- [2] M. A. Ahmad, Z. Zulkifely, and M. A. Zawawi, "Experimental investigations of input shaping schemes for sway control of a gantry crane system," *2nd Int. Conf. Comput. Netw. Technol. ICCNT 2010*, no. 1, pp. 483–486, 2010.
- [3] M. A. Ahmad, A. N. K. Nasir, M. S. Najib, and H. Ishak, "Anti-sway techniques in feedback control loop of a gantry crane system: A comparative assessment of PD and PD-type fuzzy logic controller," *2009 4th IEEE Conf. Ind. Electron. Appl. ICIEA 2009*, vol. 3, no. 3, pp. 2483–2487, 2009.
- [4] M. Ajayan and P. N. Nishad, "Vibration control of 3D gantry crane with precise positioning in two dimensions," *2014 Annu. Int. Conf. Emerg. Res. Areas Magn. Mach. Drives, AICERA/iCMMD 2014 - Proc.*, 2014.
- [5] N. B. Almutairi and M. Zribi, "Fuzzy Controllers for a Gantry Crane System with Experimental Verifications," *Math. Probl. Eng.*, vol. 2016, 2016.
- [6] E. Omid, "Modeling and Nonlinear Control of Gantry Crane Using Feedback Linearization Method," no. May, pp. 1–7, 2014.
- [7] A. Alhassan, K. A. Danapalasingam, M. Shehu, A. M. Abdullahi, and A. Shehu, "Closed-loop schemes for position and sway control of a gantry crane system," *Int. J. Simul. Syst. Sci. Technol.*, vol. 17, no. 32, pp. 1–8, 2016.
- [8] A. Cakan and U. Onen, "Position Regulation And Sway Control Of A Nonlinear Gantry Crane System," *Int. J. Sci. Technol. Res.*, vol. 5, no. 10, pp. 121–124, 2016.
- [9] Z. Feng, J. Yang, and E. Shao, "Anti-Sway Control of Underactuated Cranes Using Linear Quadratic Regulator and Extended State Observer Techniques," *Proc. 32nd Chinese Control Decis. Conf. CCDC 2020*, pp. 2893–2898, 2020.

- [10] H. Y. Zhang, J. Wang, and G. D. Lu, "Hierarchical fuzzy-tuned multiobjective optimization control for gantry cranes," *Proc. Inst. Mech. Eng. Part C J. Mech. Eng. Sci.*, vol. 228, no. 7, pp. 1119–1131, 2014.
- [11] B. Abduljabbar, J. Billingsley, and P. Wen, "A pragmatic controller for a gantry crane," *J. Phys. Conf. Ser.*, vol. 1780, no. 1, 2021.
- [12] P. A. Ospina-Henao and F. López-Suspes, "Dynamic analysis and control PID path of a model type gantry crane," *J. Phys. Conf. Ser.*, vol. 850, no. 1, 2017.
- [13] M. I. Solihin, Wahyudi, M. A. S. Kamal, and A. Legowo, "Optimal PID controller tuning of automatic gantry crane using PSO algorithm," *Proceeding 5th Int. Symp. Mechatronics its Appl. ISMA 2008*, no. June, 2008.
- [14] S. W. Su et al., "Model predictive control of gantry crane with input nonlinearity compensation," *World Acad. Sci. Eng. Technol.*, vol. 38, pp. 312–316, 2009.
- [15] N. M. Bashir, A. A. Bature, and A. M. Abdullahi, "Pole Placement Control of a 2D Gantry Crane System with Varying Pole Locations," *Appl. Model. Simul.*, vol. 2, no. 1, pp. 8–16, 2018.
- [16] A. B. Alhassan, B. B. Muhammad, K. A. Danapalasingam, and Y. Md Sam, "Optimal analysis and control of 2D nonlinear gantry crane system," *2015 Int. Conf. Smart Sensors Appl. ICSSA 2015*, pp. 30–35, 2015.
- [17] M. A. Ahmad, H. Ishak, A. N. K. Nasir, and M. S. Ramli, "Comparison of active sway control of a gantry crane system," *Proc. - 2009 3rd Asia Int. Conf. Model. Simulation, AMS 2009*, pp. 431–435, 2009.
- [18] M. A. Ahmad, A. N. K. Nasir, N. Hambali, and H. Ishak, "Hybrid input shaping and PD-type Fuzzy Logic control scheme of a gantry crane system," *Proc. IEEE Int. Conf. Control Appl.*, pp. 1051–1056, 2009.
- [19] I. Golovin, A. Maksakov, and S. Palis, "Gantry crane position control via parallel feed-forward compensator," *2020 28th Mediterr. Conf. Control Autom. MED 2020*, pp. 1045–1050, 2020.

- [20] A. Aktas, K. Bruggeman, H. Yazici, and M. Sever, “Anti-sway control of a gantry crane with LMI based robust pole placement: Experimental verification for acceleration control approach,” 2018 6th Int. Conf. Control Eng. Inf. Technol. CEIT 2018, no. October, pp. 1–6, 2018.
- [21] E. Park and C. Martinez, “Motion Design Problem for a Gantry Crane” California state University Fullerton, July 11, 2021.
- [22] M. Schleicher, “Control Engineering: Basic principles and tips for practitioners” Fulda, Germany, October 2014.
- [23] A. Alhassan, K. A. Danapalasingam, M. Shehu, A. Abdullahi, and A. Tijjani, “Closed-loop schemes for position and sway control of a gantry crane system” International Journal of Simulation Systems, Science and Technology, vol. 17, 01 2016.
- [24] A. M.A and Z. Mohamed, “Hybrid fuzzy logic control with input shaping for input tracking and sway suppression of a gantry crane system,” American Journal of Engineering and Applied Sciences, vol. 2, 01 2009.
- [25] K. Khandakji, V. Busher, and L. Melnikova, “Anti-Sway System for Container Cranes with Coordinated Motion Control and Reduced Dynamic Loads,” *JJEE Jordan J. Electr. Eng. ISSN (Print)*, vol. 3, no. 2, pp. 86–101, 2017.
- [26] K. Åström and T. Hägglund, “Advanced PID Control ISA the Instrumentation, Systems and Automation Society” 2006.
- [27] S. Moorjani, “Directing cell migration by dynamic control of laminar streams,” Ph.D. dissertation, 12 2010.
- [28] G. Chen and T. Pham, “Introduction to Fuzzy Sets, Fuzzy Logic, and Fuzzy Control Systems CRC Press 2000.
- [29] L. Zadeh, G. Klir, and B. Yuan, Fuzzy Sets, Fuzzy Logic, and Fuzzy Systems: Selected Papers, ser. Advances in fuzzy systems: applications and theory, World Scientific 1996.
- [30] Roland S. Burns, Advanced Control Engineering, A division of Reed Educational and Professional Publishing Ltd, University of Plymouth, UK, 2001

[31] Huang, Jigang & Jie, Wang & Hui, Fang, an anti-windup self-tuning fuzzy PID controller for speed control of brushless DC motor. *Automatika*. 58. 321-335. 2017.

APPENDIX I

MATLAB Code

```
%%Trapezoidal Path Planning Algorithm
```

```
clear all
```

```
close all;
```

```
clc
```

```
t0 = 0;
```

```
tf = 10;
```

```
q0 = 0;
```

```
qf = 61*pi/6;
```

```
V = 1.5*(qf-q0)/tf;
```

```
tb = (q0-qf+V*t)/V;
```

```
t = linspace(t0,t,100);
```

```
ind1 = find((t0<=t) & (t<=tb));
```

```
ind2 = find((t<t) & (t<=tf-tb));
```

```
ind3 = find((f-tb<t) & (t<=tf));
```

```
q1 = q0+V*t.^2/(2*tb);
```

```
q2 = (qf+q0-V*tf)/2+V*t;
```

```
q3 = qf-V*tf^2/(2*t)+V*t*t/tb-V*t.^2/(2*tb);
```

```
q = [q1(ind1) q2(ind2) q3(ind3)];
```

```
qdot = dif(q)./dif(t);
```

```
q2dot = dif(qdot)./dif(t(1:end-1));
```

```
% System Consitans for Simulink
```

```
m = 2.5; %pedulum mass
```

```
M = 10; %cart Mass
```

```
L = 0.4;
```

```
g = 9.8;
```

```
sim('FuzzyPID_Gantry_Crane_2DModel');
```

```
%extract data
```

```
t1 = Dis.time;
```

```
x1 = Dis;
```

```
x2 = Velocity;
```

```
x3 = Theta;
```

```

x4 = Angular_Velocity;

figure(1)
subplot(2,1,1)
plot(t,q,'b','Linewidth',2)
grid on
xlabel('t(sec)');
ylabel('Dis(m)')

%title('Trapizoial Velocity Optimal Path Planning');
subplot(2,1,2)
plot(t(1:end-1),qdot,'b','Linewidth',2)
grid on
xlabel('t(sec)');
ylabel('Velocity(m/sec)')
% subplot(3,1,3)
% plot(t(1:end-2),q2dot)
% xlabel('t(sec)');
% ylabel('q2dot (rad/sec^2)')
figure(2)
subplot(2,1,1)
plot(x1,'b','Linewidth',2)
grid on
xlabel('t(sec)');
ylabel('Displacement(m)')

%title('Cart Velocity -Path Trajectory');
subplot(2,1,2)
plot(x2,'b','Linewidth',2)
grid on
xlabel('t(sec)');
ylabel('Velocity(m/sec)')

figure(3)
subplot(2,1,1)
plot(x3,'b','Linewidth',2)
grid on

```

```
xlabel('t(sec)');  
ylabel('Theta(rad)')  
  
%title('cart Swinging Condition');  
subplot(2,1,2)  
plot(x4,'b','Linewidth',2)  
grid on  
xlabel('t(sec)');  
ylabel('Angular Velocity(Rad/sec)')
```